#### An Evaluation of the Force Reduction Factor in the Force-Based Seismic Design

Gakuho Watanabe and Kazuhiko Kawashima Tokyo Institute of Technology, O-Okayama, Meguro, Tokyo, Japan, 152-8552

### ABSTRACT

This paper presents an analysis of the force reduction factors used in the force-based seismic design of structures. The force reduction factors are evaluated based on 70 free-field ground motions. Scattering of the force reduction factors depending on ground motions and the effect of damping rations assumed in linear and nonlinear responses are clarified. A new formulation of the force reduction factors is presented.

### **KEY WORDS**

Seismic design, Forced-based design, Response modification factor, Force modification factor, Seismic response

### 1. INTRODUCTION

In the force based seismic design, it is usual to estimate the demand from a linear response of a structure by dividing it by the force reduction factor. The force reduction factor or response modification factor, which is often called q-factor or R-factor, has an important role in the estimation of design force of a structure. An early study by Newmark and Hall (1973) revealed the fact that the equal displacement assumption and the equal energy assumption provide a good estimation of the force reduction factors at long and short periods, respectively. This affected an important effect to seismic design criteria worldwide. Various researches such as Nassar and Krawinkler (1991) and Miranda and Bertero (1994) have been conducted on the force reduction factors. In particular, Miranda and Bertero provided a detailed review on the force reduction factors.

However, in spite of the importance in seismic design, less attention has been paid to large scattering of the force reduction factors depending on ground motions. Since the scattering is so large, only the mean values of the force reduction factor is not sufficient to evaluate a force reduction factor for design. Assumption of damping ratio for evaluating the linear and nonlinear responses is another important point. Although it has been general to assume the same damping ratio for the linear and nonlinear responses, it depends on how the force reduction factors are used.

This paper present an analysis on the force reduction factors based on 70 free-field ground motions. The scattering of the force reduction factors depending on ground motions and the effect of assumption of damping ratios in linear and nonlinear responses are clarified.

# 2. DEFINITION OF FORCE REDUCTION FACTOR

If one idealizes a structure in terms of a single-degree-of-freedom (SDOF) oscillator with an elastic perfect plastic bilinear hysteretic behavior as shown in Fig. 1, the force reduction



Fig. 1 Definition of Force reduction Factor

factor  $R_{\mu}$  may be defined as

$$R_{\mu}(T,\mu_{T},\xi_{EL},\xi_{NL}) = \frac{F_{R}^{EL}(T,\xi_{EL})}{F_{Y}^{NL}(T,\mu_{T},\xi_{NL})}$$
(1)

in which T: natural period,  $F_R^{EL}$  and  $F_Y^{NL}$ : maximum restoring force in an oscillator with a linear and a bilinear hysteresis, respectively,  $\mu_T$ : target ductility factor, and  $\xi_{EL}$  and  $\xi_{NL}$ : damping ratio assumed in the evaluation of linear and bilinear responses, respectively. The natural period T may be evaluated based on the cracked stiffness of columns. Representing  $u_y$  the yield displacement where the stiffness changes from the cracked stiffness to the post-yield stiffness, a target ductility factor  $\mu_T$  may be defined based on the yielding displacement  $u_y$  as

$$\mu_T = \frac{u_{\max T}}{u_y} \tag{2}$$

in which  $u_{\max T}$  is a target maximum displacement of an oscillator. The post-yield stiffness is assumed to be 0 in the present study.

Since the damping controls structural response, it has to be clarified carefully. A structure under a strong excitation generally exhibits strong hysteretic behavior, and this results in an energy dissipation in a structure. For example, the flexural inelastic deformation of columns contributes to energy dissipation in a bridge. Hence, the evaluation of damping ratio depends on the idealization of such an energy dissipation. If one idealizes the energy dissipation in nonlinear structural components by incorporating nonlinear elements that represent the hysteretic behavior, the energy dissipation in the nonlinear structural components is automatically included in the analysis. On the other hand, if one idealizes the nonlinear structural components by elastic linear elements, the energy dissipation in the nonlinear structural components has to be included in the analysis by other means. The equivalent viscous damping ratio  $\xi_h$  is generally used for such a purpose as

$$\xi_h = \frac{1}{4\pi} \cdot \frac{\Delta W}{W} \tag{3}$$

in which  $\Delta W$  and W represent an energy dissipation in a hysteretic excursion and the elastic energy, respectively. For example, in an oscillator with an elastic perfect-plastic bilinear hysteresis, the equivalent damping ratio  $\xi_h$  is

$$\xi_h = \frac{2}{\pi} \frac{\mu - 1}{\mu} \tag{4}$$

Fig. 2 shows the equivalent damping ratio by Eq. (4). It is generally very large such as 0.4 at the target ductility factor of 3-5.

In addition to such hysteretic energy dissipation, there must be some other sources of energy dissipation (for example, Kawashima, Unjoh, Tsunomoto 1993). The radiation of energy from a foundation to surround ground contributes to energy dissipation. Structural damping such as friction at connections may be important in many structures (for example, Kawashima and Unjoh 1989). Viscous damping due to friction with air is generally predominant in a structure with a long natural period. It is general to idealize those sources of energy dissipation in terms of the equivalent viscous damping.

If one considers a structure in which the flexural hysteretic energy dissipation is predominant with other sources of energy dissipation being a secondary importance, the total damping ratio  $\xi_{eq}$  of a SDOF oscillator



Fig.2 Equivalent Damping Ratio  $\xi_h$  by Eq. (4) may be provided as

 $\xi_{eq} = \xi_h + \xi_{oth} \tag{5}$  in which  $\xi_h$  is the damping ratio that accounts the hysteretic energy dissipation by Eq. (3), and  $\xi_{oth}$  is the damping ratio that accounts the energy dissipation other than the hysteretic energy dissipation.

In the evaluation of the force reduction factor  $R_{\mu}$  based on Eq. (1), how damping ratios are assumed in the evaluation of the linear and the nonlinear responses is important. If one assumes the damping ratios as

$$\xi_{EL} = \xi_{eq}$$
 and  $\xi_{NL} = \xi_{oth}$  (6)

the energy dissipation is essentially the same between the linear and the nonlinear responses. Hence, the force reduction factor by Eq. (1) represents the difference of restoring force between the linear and nonlinear responses. Thus, Eq. (1) reflects the effect of nonlinear response of an oscillator.

On the other hand, if one assumes the damping ratios as

 $\xi_{EL} = \xi_{NL} = \xi_{ea}$ 

or,

$$\xi_{EL} = \xi_{NL} = \xi_{oth} \tag{8}$$

(7)

the force reduction factor by Eq. (1) includes the effect of different energy dissipation between the linear and nonlinear responses, in addition to the effect of nonlinear response. By assuming Eq. (7) in the evaluation of nonlinear response, the hysteretic energy dissipation in the nonlinear structural components is counted by the equivalent viscous damping in addition to the inelastic excursion in the nonlinear elements. As a consequence, the hysteretic energy dissipation in the nonlinear structural components is counted twice in the evaluation of nonlinear response. On the other hand, if one assumes Eq. (8), the hysteretic energy dissipation is not taken into account in the evaluation of linear response.

It should be noted here that which is appropriate among Eqs. (6), (7) and (8) depends on how the force reduction factor is used. Based on the original definition inherent to the force reduction factor, it seems that Eq. (6) is the most appropriate. Eq. (8)generally provides conservative estimation for the force reduction factors. If  $\xi_{oth} \approx 0$ , the difference of the force reduction factors among Eqs. (6), (7) and (8) is limited. It should be noted that  $\xi_h$ ,  $\xi_{oth}$  and  $\xi_{eq}$ depend on the type of a structure, mode shape, hysteresis and the target ductility factor.

Although the equivalent damping ratio  $\xi_h$  is very high as shown in Fig. 2, it is not general to assume such a high damping ratio in seismic design of a bridge structure. It is because a bridge structure is generally more complex than a SDOF oscillator, and this makes the relative contribution of the hysteretic energy dissipation of columns less significant. Since it is general practice in a standard bridge structure to assume about 0.05 for the damping ratio including hysteretic energy dissipation of columns,  $\xi_{eq}$  is assumed to be 0.05 in the present study based on Eq. (6). Hence, it is assumed here that  $\xi_h$  and  $\xi_{oth}$  is 0.03 and 0.02, respectively.

those damping ratios, the force Using reduction factors are evaluated in this study based on Eq. (6). However, an analysis assuming Eqs. (7) and (8) is also conducted for comparison with the previous studies.

## 3. REVIEW OF THE PAST INVESTIGATIONS

An early study for the force reduction factor was conducted by Newmark and Hall (Newmark and Hall 1973). They used 10 ground motions recorded in the 1940 Imperial Valley Earthquake. They assumed  $\xi_{EL} = \xi_{NL} = 0.05$ , and proposed a force reduction factor as

$$R_{\mu} = \begin{cases} 1 \cdots (0 \le T \le T_{1}/10) \\ \sqrt{2\mu - 1} \left(\frac{T}{4T_{1}}\right)^{2.513 \log(1/\sqrt{2\mu - 1})} \\ \cdots (T_{1}/10 \le T \le T_{1}/4) \\ \sqrt{2\mu - 1} \cdots (T_{1}/4 \le T \le T_{1}') \\ T\mu/T_{1} \cdots (T_{1}' \le T \le T_{1}) \\ \mu \cdots (T_{1} \le T \le T_{2}) \end{cases}$$

where,

$$T_{1} = 2\pi \frac{\phi_{ev}V}{\phi_{ea}A}$$

$$T_{1}' = T_{1} \frac{\mu}{\sqrt{2\mu - 1}}$$

$$T_{1} = 2\pi \frac{\phi_{ed}D}{\phi_{ev}V}$$
(10)

in which, A, V and D represent peak ground acceleration, velocity and displacement, respectively, and  $\phi_{ea}$ ,  $\phi_{ev}$  and  $\phi_{ed}$  represent the amplification for acceleration, velocity and displacement, respectively.

Nassar and Krawinkler proposed a force reduction factor, assuming  $\xi_{EL} = \xi_{NL} = 0.05$ , based on an analysis for 15 ground motions as (Nassar and Krawinkler 1991)

$$R_{\mu} = \{c(\mu - 1) + 1\}^{1/c}$$
(11)

where

$$c(T,\alpha) = \frac{T^{\alpha}}{1+T^{\alpha}} + \frac{b}{T}$$
(12)

in which  $\alpha$  represents a ratio of the post-yield

stiffness to the initial elastic stiffness, and a and b are coefficients depending on  $\alpha$ . Nassar and Krawinkler precisely analyzed the effect of stiffness deterioration, and provided the coefficients a and b depending on  $\alpha$ .

Miranda and Bertero proposed a force reduction factor, assuming  $\xi_{EL} = \xi_{NL} = 0.05$ , based on an analysis for 124 ground motions as (Miranda and Bertero 1994)

$$R_{\mu} = \frac{\mu - 1}{\Phi(T, T_g)} + 1 > 1$$
(13)

where,

(9)

$$\Phi = \begin{cases} 1 + \frac{1}{10T - \mu T} - \frac{1}{2T} \exp\left\{-\frac{3}{2}\left(\ln T - \frac{3}{5}\right)^{2}\right\} \\ \dots \dots \dots (\operatorname{rock}) \\ 1 + \frac{1}{12T - \mu T} - \frac{2}{5T} \exp\left\{-2\left(\ln T - \frac{1}{5}\right)^{2}\right\} \\ \dots \dots (\operatorname{alluvium}) \\ 1 + \frac{T_{g}}{3T} - \frac{3T_{g}}{4T} \exp\left\{-3\left(\ln \frac{T}{T_{g}} - \frac{1}{4}\right)^{2}\right\} \\ \dots \dots (\operatorname{soft}) \end{cases}$$

$$(14)$$

in which  $T_g$  represents a most predominant period.

It has been known that the equal energy assumption provides a good estimation for the force reduction factor at short periods while the equal displacement assumption at long periods. The force reduction factor provided by the equal energy and the equal displacement assumptions are given as

$$R_{\mu} = \sqrt{2\mu - 1} \quad \text{(equal energy)} \quad (15)$$
  
$$R_{\mu} = \mu \quad \text{(equal displacement)} \quad (16)$$

Application of Eqs. (15) and (16) and a comparison of the present study to the previous models will be described later.

## 4. FORCE REDUCTION FACTOR FOR BILINEAR OSCILLATORS

Force reduction factors were evaluated for target ductility factor  $\mu_T$  of 2, 4, 6 and 8 assuming an elastic perfect-plastic bilinear hysteresis. Damping ratio in the linear and nonlinear analyses is assumed as  $\xi_{EL} = 0.05$  and  $\xi_{NL} = 0.02$  based on Eq. (6). Seventy free field ground accelerations by 64 shallow earthquakes with depth less that 60 km were used for analysis. They are classified into three soil conditions depending on the fundamental natural period of subsurface ground  $T_g$ ; stiff ( $T_g < 0.2$  s), moderate ( $0.2 \le T_g < 0.6$  s) and soft  $(T_g \ge 0.6 \text{ s})$  (Japan Road Association 2002). Number of records in the stiff, moderate and soft categories is 16, 39 and 15, respectively. Distribution of peak ground accelerations on the earthquake magnitudes and epicentral distances is shown in Fig. 3. The peak accelerations are in the range of 0.1-8 m/s<sup>2</sup>, and the epicentral distances are in the range of 10-500 km.

Fig. 4 shows the force reduction factors for the 70 ground motions. Only the results for  $\mu_T=4$ and 6 are presented here since the results for other target ductility factors show the similar characteristics. It is seen in Fig. 4 that scattering of the force reduction factors depending on ground motions is significant. For example at natural period of 1 second, the force reduction factors varies from 1.9 to 10.3 depending on ground motions for  $\mu_T = 4$  at the moderate soil sites. It is apparent that such a large scattering of the force reduction factors result in a large change of sizing of a structure in seismic design. Obviously smaller force reduction factors should be assumed in design to provide conservative design. It is observed in Fig. 4 that the dependence of force reduction factors on the soil condition is less significant. This will be discussed later.

Since the scattering of the force reduction



Fig. 3 Classification of Ground Accelerations in Terms of Soil Conditions and Earthquake Magnitudes

factors depending on ground motions is so large that the means +/- one standard deviations of the force reduction factors were obtained for each target ductility factor, natural period and soil condition. Fig. 5 shows the mean values and the mean values +/- one standard deviations of the force reduction factors presented in Fig. 4. The force reduction factors predicted by Eqs. (15) and (16) based on the equal displacement and the equal energy assumptions are also presented here for comparison. The mean values of force reduction factors increase as the natural periods increase, and then they approach to  $\mu_T$  at long period. It has been pointed out in the previous researches that the Eq. (15) provides a good estimation to the force reduction factor. However, it is noted that Eq. (15) provides a good estimation the mean values, but it considerably to underestimates the force reduction factors corresponding to the mean values minus one standard deviations. On the other hand, Eq. (16)



provides better estimation to the mean values minus one standard deviations. Taking account of the force reduction factors having considerable scattering depending on ground motions, it seems reasonable to consider a certain redundancy in the estimation of the force reduction factor in design.



Fig. 5 Mean and Mean +/- One Standard Deviation of the Force Reduction Factors for 70 Ground Motions

Based on such a consideration, it is more conservative to assume Eq. (16) instead of Eq. (15) for a design purpose.

Fig. 6 shows the dependence of the standard deviations of force reduction factors  $\sigma(R_{\mu})$  on the natural periods T and the soil condition. Similar to the mean values, the standard deviations  $\sigma(R_{\mu})$  increase as the natural periods increase, and decrease after taking peak values at natural period of 1-2 second. Fig. 7 shows the dependence of the standard deviations  $\sigma(R_{\mu})$  on the target ductility factors  $\mu_T$ . The standard

deviations  $\sigma(R_{\mu})$  increase as the target ductility factors increase. The relation may be approximated by a least square fit as

$$\sigma(R_{\mu}) = \begin{cases} -0.328 + 0.379 \cdot \mu_{T} & \text{(stiff)} \\ -0.292 + 0.378 \cdot \mu_{T} & \text{(moderate)} & (17) \\ -0.354 + 0.409 \cdot \mu_{T} & \text{(soft)} \end{cases}$$

As the soil condition dependence of  $\sigma(R_{\mu})$  is less significant as shown in Fig. 7, Eq. (17) may be approximated as

$$\sigma(R_{\mu}) \approx -0.3 + 0.4 \cdot \mu_T \tag{18}$$



Fig. 6 Natural Period Dependence of Standard Deviations of the Force Reduction Factors



Fig. 7 Target Ductility Factor Dependence of the Standard Deviations of Force reduction Factors

## 5. FORMULATION OF FORCE REDUCTION FACTORS

To idealize the mean values of the force reduction factors in Fig. 5, they are represented as

$$R_{\mu} = (\mu - 1) \cdot \Psi(T) + 1$$
 (19)



Fig. 8 Idealization of Force Reduction Factors

where,

$$\Psi(T) = c \cdot \frac{T-a}{e^{b \cdot (T-a)}} + 1 \tag{20}$$

in which a, b and c are parameters to be determined.

Since  $R_{\mu} = \mu$  at T = a in Eq. (20), the parameter *a* represents the period where  $R_{\mu}$  is equal to  $\mu$  (Point P) as shown in Fig. 8. Because the gradient of  $R_{\mu}$  is

$$\frac{dR_{\mu}}{dT} = c(\mu - 1) \cdot \frac{1 - b(T - a)}{e^{b(T - a)}}$$
(21)

it is  $c \cdot (\mu - 1)$  at Point P. Consequently, the parameter *c* represents the gradient at Point c divided by  $\mu - 1$ . Representing Q as the point where  $R_{\mu}$  takes the peak value, 1/b represents the period between Points P and Q.

Based on the definition, the following condition has to be satisfied in  $R_{\mu}$ 

$$\lim_{T \to 0} R_{\mu} = 0 \tag{22}$$

Hence, the coefficient c can be eliminated as  $c = 1/ae^{ab}$  (23)

Substitution of Eq. (23) makes Eq. (20) as

$$\Psi(T) = \frac{T-a}{ae^{bT}} + 1 \tag{24}$$

It is noted that Eq. (19) automatically satisfies the following condition

$$\lim_{T \to \infty} R_{\mu} = \mu \tag{25}$$

Table 1 Parameters *a* and *b* and Regression Coefficients *R* ( $\xi_{NL}$ =0.02 and  $\xi_{EL}$ = 0.05)

$\eta_T$	<i>a</i> , <i>b</i>	Soil Conditions		
	and R	Type-I	Type-II	Type-III
2	а	1.29	1.12	2.35
	b	2.77	2.18	1.69
	R	0.379	0.701	0.851
4	а	1.24	0.989	1.52
	b	2.39	1.62	1.05
	R	0.673	0.842	0.886
6	а	1.34	1.03	1.85
	b	2.15	1.24	0.821
	R	0.717	0.869	0.878
8	а	1.36	1.20	1.74
	b	1.67	1.11	0.611
	R	0.776	0.899	0.895

It is a feature of the above formulation that the equation is simpler and the physical meaning of the parameters a and b is clearer than the previous studies.

The mean values of force reduction factors in Fig. 5 were fitted by Eq. (19) using a nonlinear least square method (Press et al 1996). Table 1 represents the *a* and *b* as well as the regression coefficients. Although the regression coefficient is not high enough for some combinations such as  $\mu_T$ =2 and stiff sites, it may be accepted in other conditions. As shown later, the fitting is not necessarily poor for a combination of  $\mu_T$ =2 and stiff sites.

Fig. 9 shows parameters a, 1/b and a+1/b. Parameter a is in the range of 1.0-1.4 second at stiff and moderate sites, and 1.5-2.4 second at soft sites. They are less sensitive to the target ductility factor  $\mu_T$  between 2 and 8. As described before, a represents the period where  $R_{\mu} = \mu$ , which implies that the equal displacement assumption by Eq. (16) provides the best estimation at period a. Consequently, the accuracy of equal displacement assumption is high at 1.0-1.4 second at stiff and



Fig. 9 Parameters *a* and a+1/b in Eqs. (20)



Fig. 10 Natural Periods where Force Reduction Factors Take Values Predicted by the Equal Energy Assumption (Eq. (15))

moderate sites, and 1.5-2.4 second at soft site.

As shown in Fig. 8, a+1/b represents the natural period where  $R_{\mu}$  takes the peak value. It is 1.5-2 second at stiff and moderate sites, and



Fig. 11 Application of Eq. (19) to the Mean Force Reduction Factors Presented in Fig. 5

2.5-3.5 second at soft site. It slightly increases as target ductility  $\mu_T$  increases.

The natural periods where  $R_{\mu}$  take the values predicted by Eq. (15) based on the equal energy assumption are obtained as shown in Fig. 10. They are in the range of 0.2-0.36 second, 0.26-0.4 second and 0.4-0.6 second at stiff, moderate and



Fig. 12 Effect of Soil Condition on the Force Reduction Factors Predicted by Eq. (19)

soft sites, respectively. They are much shorter than the natural periods where the equal displacement assumption provides the best approximation.

Fig. 11 compares the mean force reduction factors presented in Fig. 5 to the values predicted by Eq. (19). Although some discrepancies are observed at larger target ductility factors, Eq. (19)



Fig.13 Force Reduction Factors Corresponding to Means minus One Standard Deviations

provides a good estimation for the mean force reduction factors.

Fig. 12 shows the effect of soil condition on the mean force reduction factors estimated by Eq. (19). The effect of soil condition is less significant on the force reduction factors, in particular at small target ductility factors.

As shown in Fig. 4, scattering of the force reduction factors around the mean values is extensive. Hence, the force reduction factors corresponding to the mean values m substituted by a standard deviation  $\sigma(R_{\mu})$  are evaluated as shown in Fig. 13. The mean and the standard deviation of force reduction factors were evaluated by by Eq. (19) and Eq. (18), respectively, in this estimation. They are of course

Table 2 Parameters *a* and *b* ( $\xi_{NL} = \xi_{EL} = 0.02$ )

$\mu_T$	<i>a</i> and	Soil Conditions		
	b	Type-I	Type-II	Type-III
2	а	0.152	0.225	0.361
	b	0.289	1.60	1.12
4	а	0.289	0.348	0.600
	b	2.46	1.28	0.902
6	а	0.397	0.432	0.800
	b	1.81	1.14	0.768
8	а	0.507	0.513	0.916
	b	1.14	1.04	0.632

Table 3 Parameters *a* and *b* ( $\xi_{NL} = \xi_{EL} = 0.05$ )

$\mu_T$	<i>a</i> and	Soil Conditions		
	b	Type-I	Type-II	Type-III
2	а	0.226	0.344	0.521
	b	4.14	1.94	1.34
4	а	0.778	0.572	0.976
	b	3.50	1.35	0.994
6	а	0.981	0.725	1.23
	b	2.93	1.15	0.757
8	а	1.23	0.807	1.28
	b	2.57	0.983	0.569

close to the force reduction factors of the mean minus one standard deviation directly computed from the 70 ground motions (refer to Fig. 5). The force reduction factors predicted by Eq. (15) based on the equal energy assumption are presented here for comparison. From Fig. 13, it is seen that at  $\mu_T = 4$ , the equal energy assumption provides a good estimation at natural periods longer than 0.5 second at stiff and moderate sites and 1.2 second at soft sites, while it provides underestimation at natural periods shorter than those values. On the other hand, at  $\mu_T = 8$ , the equal energy assumption provides a good estimation at 0.6 second at stiff and moderate sites and 1 second at soft sites. It underestimates and reduction overestimates the force factors



Fig. 14 Dependence of Parameters a and a+1/b on the Assumption of Damping Ratios

corresponding to the mean minus one standard deviation at natural periods shorter and longer, respectively, than the above natural periods.

## 6. EFFECT OF DAMPING RATIOS

In the preceding analysis, the force reduction factors were evaluated based on Eq. (1) assuming  $\xi_{EL} = 0.05$  and  $\xi_{NL} = 0.02$ . However in the past researches, damping ratios were usually assumed as  $\xi_{EL} = \xi_{NL} = 0.05$ . Consequently, the same analysis presented in the preceding chapters was conducted by assuming  $\xi_{EL} = \xi_{NL} = 0.05$  based on Eq. (7) using the same ground motion data set.

For comparison, an analysis was also conducted assuming  $\xi_{EL} = \xi_{NL} = 0.02$  based on Eq. (8).

Tables 2 and 3 show the parameters a and bdetermined for a combination of  $\xi_{EL} = \xi_{NL} = 0.02$ and  $\xi_{EL} = \xi_{NL} = 0.05$ , respectively. Fig. 14 compares a and a+1/b thus determined. Also presented in Fig. 14 are a and a+1/b used in the preceding chapter ( $\xi_{EL}$  =0.05 and  $\xi_{NL}$  =0.02, refer to Fig. 9). It is seen in Fig. 14 that both a and a+1/b at the same target ductility factors are the shortest for а combination of  $\xi_{EL} = \xi_{NL} = 0.02$  and the longest for a combination of  $\xi_{EL}$  =0.05 and  $\xi_{NL}$  =0.02. Parameters *a* and a+1/b for a combination of  $\xi_{EL} = \xi_{NL} = 0.05$  are between the two cases.



Fig. 15 Dependence of Force Reduction Factors on the Assumption of Damping Ratios

Fig. 15 compares the mean values of the force reduction factors based on the three assumptions of damping ratios. Original force reduction factors computed from the 70 ground motions are also presented here for comparison. A systematic difference of the force reduction factors is observed reflecting the dependence of *a* and a+1/b on the damping rations. The combination

of  $\xi_{EL} = \xi_{NL} = 0.02$  provides the largest estimation for the force reduction factors, while the combination of  $\xi_{EL} = 0.05$  and  $\xi_{NL} = 0.02$ provides the smallest estimation. The combination of  $\xi_{EL} = \xi_{NL} = 0.05$  provides the estimation between the two cases.



Fig. 16 Comparison with Previous Studies

# 7. COMPARISON WITH THE PREVIOUS STUDIES

Fig. 16 shows a comparison of the force reduction factor in the present study by Eq. (19) to Nassar and Krawinkler by Eq. (11) and Miranda and Bertero by Eq. (13). Since it is assumed in Eqs. (11) and (13) that  $\xi_{EL} = \xi_{NL} = 0.05$ , the same

damping ratios are assumed in the present study for comparison. The original mean values of the force reduction factors computed from the 70 ground motions are also presented here for comparison. It is noted that definition of soil conditions is not the same among three researches. Hence they are classified into stiff, moderate and soft. In the Miranda and Bertero formulation,  $T_g$  was assumed 1.5 second at soft (alluvial) site in Eq. (14).

From Fig. 16, it is seen that the present study provides a quite similar result to the formulations by Miranda & Mertero and Nassar & Krawinkler if the same damping ratios are assumed in the evaluation of linear and nonlinear responses.

## 8. CONCLUSIONS

An analysis was conducted for the force reduction factor based on response of SDOF oscillator using 70 free-field ground motions. Based on the analysis presented herein, the following conclusions may be deduced:

1) A new formulation as shown in Eqs. (19) and (24) was developed. The formulation is simpler than the past formulations. Parameters *a* and a+1/b express the natural period where  $R_{\mu}$  is equal to  $\mu$  and  $R_{\mu}$  takes a peak value, respectively.

2) Difference of the damping ratios assumed in the evaluation of linear and nonlinear responses  $(\xi_{EL} \text{ and } \xi_{NL})$  provides a systematic difference in the force reduction factors. The combination of  $\xi_{EL} = \xi_{NL} = 0.02$  provides the largest estimation for the force reduction factors, while the combination of  $\xi_{EL} = 0.05$  and  $\xi_{NL} = 0.02$ provides the smallest estimation. The combination of  $\xi_{EL} = \xi_{NL} = 0.05$  provides the estimation between the two cases. Hence, the damping ratios have to be carefully assumed keeping how the force reduction factors are used in mind.

3) Scattering of the force reduction factors depending on ground motions is significant. Although it has been pointed out that the equal displacement assumption by Eq. (15) provides a good estimation to the force reduction factors, it provides a good estimation only to the mean values; however, it considerably underestimates

the mean minus one standard deviation. On the other hand, the equal energy assumption by Eq. (16) provides a better estimation to the force reduction factors corresponding to the mean minus one standard deviation, although it provides too conservative estimation to the mean values. Taking account of the considerable scattering of the force reduction factors depending on ground motions, it is conservative to assume the equal energy assumption instead of the equal displacement assumption.

4) The response modification factors in the present study by Eqs. (19) and (24) provides quite close force reduction factors proposed by Nassar and Krawinkler, and Miranda and Betero, if the damping ratios are assumed as  $\xi_{EL} = \xi_{NL} = 0.05$ .

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