TIME-FREQUENCY TRANSFER FUNCTIONS AND IDENTIFICATION OF PILE HEAD IMPEDANCE FROM EXPERIMENTAL DATA

Roger GHANEM and Samar MALEK
The Johns Hopkins University, Baltimore, MD 21218, USA.

ABSTRACT

This paper presents results from the analysis of experimental data related to the dynamic behavior of pile groups. By idealizing the structure-soil system as a three degree of freedom model, equations are obtained and implemented for the impedance function of the soil-pile system. Data obtained from a series of shaking table experiments, are utilized for estimating the impedance functions of a soil-pile foundation. The data corresponds to a scaled model of a superstructure supported by a foundation consisting of a pile cap and nine piles. Measurements of both the far field and the near field motions are obtained. Furthermore, the experiment is repeated for varying levels of ground shaking magnitudes in order to capture the nonlinear character of the soil-pile-structure dynamics. A least squares procedure is developed for estimating the impedance functions of the soil-pile system. The procedure relies on the frequency dependence of the impedance functions being time-invariant. The correctness of this postulate is investigated through a joint time-frequency analysis of various transfer functions involved in the evaluation of the impedance functions. It is found that these transfer functions are somewhat sensitive to the length of the analyzing window in a moving-window Fourier analysis. It is found that the peaks of the transfer functions, as well as their frequency spread, is dependent on the length of this window, in general resulting in a wider spread and a smaller amplification with the smaller windows. This suggests that the parameters of the superstructure should be taken into consideration in deciding on an appropriate window size.

Keywords: Pile Dynamics, Experimental Data, System Identification, Cross-Spectral Analysis, Time-Frequency Analysis.

INTRODUCTION

A number of phenomena are involved in the response of pile-supported structures to dynamic loads. Conceptually, the picture is rather simple, consisting of the load, whether it is applied on the pile head or via a ground shaking at the level of the bedrock, propagating along the pile and through the medium. The motion of the medium then excites the pile further which in turn scatters more waves causing additional motion in the medium, and leading to medium-pile interaction. The presence of other piles or inclusions either in the medium or on its boundary causes additional scattering which further affects the displacement field in the medium. The vibration of the pile is constrained by the pile cap which couples its motion to that of an overlaying superstructure or to the motion of neighboring piles tied to the same pile cap. To complete the picture, it should be noted that the medium in question is soil, and in many cases of interest, and in particular the focus of the present paper, the applied load is due to a strong ground motion caused by an earthquake. Thus the problem of analysing the dynamic response of a pile or pile group to ground motion can be restated as that of characterizing the propagation of nonstationary waves in a medium that reflects soil's response to wave passage.

In this paper, the assumption of time invariance, implicit in all frequency domain analyses, is tested and its significance as related to the calculation of impedance functions from measured data investigated. Lack of time invariance can be attributed to a number of phenomena chief among which are damping, material nonlinearity, and contact behavior between the pile and the soil. Data form an experiment conducted at the Shimizu Institute of Technology in Tokyo, Japan, is used for that purpose. A procedure is also developed that allows the identification of all the components of the impedance matrix.

In view of the nonlinear nature of soil dynamics, and its pronounced dependence on loading amplitude, developing and refining the concept of the impedance function is of great value. This paper reports on the processing of data obtained in the course of a laboratory experiment involving pile groups subjected to various types of excitations. The experiment was well instrumented. The data collected in the course of the experiment is used to probe a number of questions associated with evaluating and understanding the impedance function and its dependence on various field measurements. Spectral analysis techniques are used to process the experimental data to obtain various transfer functions of interest along with the impedance functions. Specifically, the dependence of the impedance function on the amplitude of the excitation is addressed both by using frequency domain techniques under different input motions, as well as, by relying on time domain parameter estimation techniques that identify the time dependence of the various parameters of interest.

In the next section, the experimental set-up is described. Following that, equations to evaluate the impedance function are derived. This is followed by a presentation of the numerical results and conclusions.

EXPERIMENTAL SET-UP

The experiment was conducted at the structural laboratory of the Institute of Technology, Shimizu Corporation, in Japan. It consisted of a sand box of dimensions 1.2mx0.8m and height 1m. The outer size of the box was 1.3mx0.9 m as shown in Figure (1). A group of 9 (nine) 3 cm diameter aluminum piles were embedded in the sand. The pile thickness was 1mm, and the piles were 7.5 cm center to center, with a clearance between piles of 4.5 cm. A scale model of a one story structure was erected on top of the pile cap. The fundamental frequency of the structure with its base fixed was 24.5 Hz. An actuator connected to the base of the box provided the external driving force. The sand box consisted of layers of steel frames that could freely slide on top of each others, thus reproducing the shear mode deformations of the ground. Five accelerometers in the longitudinal direction were placed at different depths into the sand. Accelerometers were also placed on the pile cap and on the structural model. Figure (1) shows the set-up of the experiment along with the the instrumentation. Strains and displacement measurements are not shown. In this figure, labels "A" refer to measured accelerations.

The sand box was subjected to a number of different excitations. These consisted of sinusoidal inputs of magnitudes 25, 100, and 200 gal, respectively, as well as to a microtremor, and to two earthquake records with peak accelerations equal to 89 gal and 268 gal, respectively. The experiment is described in greater detail elsewhere [5].

Table (1) shows the values of the shear wave velocity (m/s) calculated during the experiment. These values were obtained by varying the shear wave velocity in a numerical algorithm until the peak of the transfer function between each accelerometer and the base matched between the numerical procedure and the measured data. In this table, the column indicating the layer number also indicates the physical extent of that layer delineated by two accelerometers.

In evaluating the impedance function of the pile group, the kinematic interaction between the pile and the soil must be evaluated. This interaction determines the true ground motion to which the structure is subjected, as it is typically different from the free-field motion. Its evaluation requires knowledge of the motion of the pile head in the absence of the superstructure. The constant amplitude tests described above were run without the superstructure. For the earthquake input experiments, data is only available in the presence of the superstructure. The transfer function, η_{eff}, of the accelerations between the free field and the pile cap was computed from the harmonic loading.

SIGNAL PROCESSING

The transfer function, $\mathcal{H}(\omega)$, of a linear system can be estimated from associated input-output pairs, $(x(t),y(t))$, according to the following formula,

Layer	Levels of Input Motion			
	Micro-Tremor	25 Gal	100 Gal	200 Gal
1 (AG2-AG1)	30	15	10	10
2 (AG3-AG2)	35	20	15	10
3 (AG4-AG3)	60	40	25	15
4 (AG5-AG4)	80	50	35	20
5 (AG6-AG5)	80	50	35	25

Table 1: Shear Wave Velocity in Different Layers Under Various Excitations.

$$\mathcal{H}(\omega) = \frac{S_{xy}(\omega)}{S_{xx}(\omega)} = |\mathcal{H}(\omega)|^2 e^{-i\phi(\omega)} \ , \tag{1}$$

where S_{xx} denotes the auto-spectrum of the input signal and S_{xy} is the cross-spectrum between the input and output. Most commonly, the Fourier transform of the signal is used in generating these spectra. In the present analysis, however, time series techniques are used for that purpose. Specifically, vector ARMA models are first fitted to each input-output pair of signals [2], following which closed forms expressions are evaluated for the spectra in terms of the coefficients of the ARMA model. An ARMA model is essentially a regression of the data on its past over a finite horizon. This procedure for estimating the transfer function has the benefit of eliminating the spurious peaks typically observed in spectra associated with the Fourier transform, and known to result in unbiased estimates [1]. One important quantity in spectral estimation, used for assessing the suitability of a certain linear model, is the coherence function, $\gamma_{xy}(\omega)$, defined as,

$$\gamma^2_{xy}(\omega) = \frac{|S_{xy}(\omega)|^2}{S_{xx}(\omega) S_{yy}(\omega)} \ . \tag{2}$$

Clearly, the coherence function takes values between 0 and 1 and it can be interpreted as the fraction of the output spectrum which is linearly caused by the input signal at frequency ω. A coherence function less than unity is usually indicative of either extraneous noise in the measurement, resolution bias errors in the spectral estimates, or nonlinearity in the system. Of course the possibility of some extraneous input is theoretically possible, but can be safely discarded in the present case. The magnitude of the transfer function is related to the coherence function through the relationship,

$$|\mathcal{H}(\omega)|^2 \ = \ \gamma_{xy} \frac{S_{yy}}{S_{xx}} \ . \tag{3}$$

It can also be shown [1] that the time delay, at frequency ω between two signal is related to the phase of the associated transfer function

$$\tau(\omega) = \frac{\phi(\omega)}{2\pi\omega} \ . \tag{4}$$

This result can be used to estimate the velocity of propagation of the signal over a distance d,

$$V_s(\omega) = \frac{2\pi\omega d}{\phi(\omega)} \ . \tag{5}$$

This expression can be used to estimate the shear wave velocity in the medium. This approach does not account for radiation and other energy dissipation mechanisms in the medium between the two signals. The coherence function can be used to estimate the validity of this assumption at various frequencies.

EQUATIONS OF MOTION

The experimental set-up is idealized, as shown in Figure (2), as a three-degree-of-freedom system [3]. The springs at the base of the model represent the impedance of the foundation and reflect its effect on the structure. Assuming a viscoelastic model for the soil medium, the impedance springs are a function of the frequency of vibration. Using m_1 to denote the mass of the superstructure, k_1 to denote its stiffness, H_1 to denote its height, and $\ddot{U}_1$ to denote the Fourier amplitude of its acceleration, the following equations of motion, expressed in the Fourier domain, are obtained,

$$m_1\ddot{U}_1 + \frac{1}{\omega^2}k_1(\ddot{U}_1 - \ddot{U}_0 - H_1\ddot{\theta}_0) = 0 \, , \tag{6}$$

$$m_0\ddot{U}_0 - \frac{1}{\omega^2}k_1(\ddot{U}_1 - \ddot{U}_0 - H_1\ddot{\theta}_0) + \frac{1}{\omega^2}k_{HH}(\ddot{U}_0 - \ddot{U}_{eff}) + \frac{1}{\omega^2}k_{H\theta}(\ddot{\theta}_0 - \ddot{\theta}_{eff}) = 0. \tag{7}$$

and

$$m_1H_1\ddot{U}_1 + (I_0 + I_1)\ddot{\theta}_0 + \frac{1}{\omega^2}k_{\theta H}(\ddot{U}_0 - \ddot{U}_{eff}) + \frac{1}{\omega^2}k_{\theta\theta}(\ddot{\theta}_0 - \ddot{\theta}_{eff}) \ = \ 0. \tag{8}$$

In these equations, m_0 denotes the mass of the footing, $\ddot{U}_0$ the Fourier amplitude of its acceleration. Furthermore, U_{eff} and θ_{eff} are the translation and rotation components, respectively, of the Fourier amplitudes of the acceleration of the effective input motion. This is the motion at the base of the structure associated with the kinematic loading, and is typically assumed to be equal to the free field motion [4]. Moreover, k_H, k_θ, and $k_{H\theta}$ denote the components of the impedance matrix in the translational, rotational, and coupling modes, respectively. Finally, I_0 and I_1 denote the moments of inertia of the superstructure and the footing, respectively, and ω denotes frequency. Implicit in these equations is the assumption that the system is linear. In what following, the effective translational motion, U_{eff}, of the ground, will be assumed to be equal to the free field motion as measured by accelerometer number AG1, while the effective rotation, θ_{eff}, will be taken as zero. Eliminating the stiffness of the superstructure, k_1, from the above equations, results in,

$$m_0\ddot{U}_0 + m_1\ddot{U}_1 - \frac{1}{\omega^2}k_{HH}(\ddot{U}_0 - \ddot{U}_{eff}) - \frac{1}{\omega^2}k_{H\theta}(\ddot{\theta}_0 - \ddot{\theta}_{eff}) \ = \ 0, \tag{9}$$

and

$$m_1H_1\ddot{U}_1 + (I_0 + I_1)\ddot{\theta}_0 - \frac{1}{\omega^2}k_{\theta H}(\ddot{U}_0 - \ddot{U}_{eff}) - \frac{1}{\omega^2}k_{\theta\theta}(\ddot{\theta}_0 - \ddot{\theta}_{eff}) \ = \ 0. \tag{10}$$

Solving the above equations and rearranging, the following expressions are obtained for the components of the pile-head impedance matrix,

$$k_{HH} = \omega^2 \frac{m_0\frac{\ddot{U}_0}{\ddot{U}_{eff}} + m_1\frac{\ddot{U}_1}{\ddot{U}_{eff}}}{\frac{\ddot{U}_0}{\ddot{U}_{eff}} - 1} - \frac{k_{H\theta}(\frac{\ddot{\theta}_0}{\ddot{U}_{eff}} - \frac{\ddot{\theta}_{eff}}{\ddot{U}_{eff}})}{\frac{\ddot{U}_0}{\ddot{U}_{eff}} - 1} \tag{11}$$

$$k_{\theta\theta} = \omega^2 \frac{m_1H_1\frac{\ddot{U}_1}{\ddot{U}_{eff}} + (I_0 + I_1)\frac{\ddot{\theta}_0}{\ddot{U}_{eff}}}{\frac{\ddot{\theta}_0 - \ddot{\theta}_{eff}}{\ddot{U}_{eff}}} - \frac{k_{\theta H}(\frac{\ddot{U}_0}{\ddot{U}_{eff}} - 1)}{\frac{\ddot{\theta}_0 - \ddot{\theta}_{eff}}{\ddot{U}_{eff}}} \tag{12}$$

Figure (3) shows the real part of the impedance function obtained using equation (11), associated with values of the cross-impedance functions, $k_{\theta H}$ and $k_{H\theta}$ equal to zero. Figure (4) shows $(\omega.\Im[k_{HH}])$, where the symbol $\Im$ denotes the imaginary part of its argument. This quantity represents radiation damping in the soil medium, In the case of the earthquake input, data is only available in the presence of the superstructure. This precludes the measurement of $\ddot{U}_{eff}$, since the collected data features interaction with the superstructure. This problem can be partially remedied by using $\ddot{U}_{gs}$, which refers to the data collected just below the ground surface, away from the pile cap, thus simulating the free field motion. A filter was designed to transmit this data to the pile cap. An approximation to this filter was obtained by analyzing the constant amplitude data, and calculating the transfer function between the free field motion and the pile cap motion in that case. The effect of the filter on the predicted results was very noticeable, and it was deduced that data from the two experiments should not be mixed. In the subsequent results, η_{eff} is taken equal to 1 over all frequencies.

LEAST SQUARES PROCEDURES

A procedure is now developed for the estimation of the impedance functions while accounting for the effect of the coupling between the translational and the rocking motions. At the same time, the procedure permits the inquiry into the time-invariance of the problem under consideration. The proposed solution consists essentially of performing a moving window frequency domain analysis followed by a least squares estimation of all the functions in the impedance matrix. Equations (11) and (12) are evaluated at every frequency and rewritten as,

$$\begin{bmatrix} (\frac{\ddot{U}_0}{\ddot{U}_{eff}} - 1) & \frac{\ddot{\theta}_0 - \ddot{\theta}_{eff}}{\ddot{U}_{eff}} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} k_{HH} \\ k_{H\theta} \end{bmatrix} = \omega^2 \left(m_0 \frac{\ddot{U}_0}{\ddot{U}_{eff}} + m_1 \frac{\ddot{U}_1}{\ddot{U}_{eff}} \right) . \tag{13}$$

and

$$\begin{bmatrix} \frac{\ddot{\theta}_0 - \ddot{\theta}_{eff}}{\ddot{U}_{eff}} & (\frac{\ddot{U}_0}{\ddot{U}_{eff}} - 1) \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} k_{\theta\theta} \\ k_{\theta H} \end{bmatrix} = \omega^2 \left(m_1 H_1 \frac{\ddot{U}_1}{\ddot{U}_{eff}} + (I_0 + I_1) \frac{\ddot{\theta}_0}{\ddot{U}_{eff}} \right) , \tag{14}$$

respectively. These last two equations can be combined, as follows,

$$\begin{bmatrix} \frac{\ddot{\theta}_0 - \ddot{\theta}_{eff}}{\ddot{U}_{eff}} & (\frac{\ddot{U}_0}{\ddot{U}_{eff}} - 1) & 0 & 0 \\ 0 & 0 & (\frac{\ddot{U}_0}{\ddot{U}_{eff}} - 1) & \frac{\ddot{\theta}_0 - \ddot{\theta}_{eff}}{\ddot{U}_{eff}} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} k_{HH} \\ k_{H\theta} \\ k_{\theta\theta} \\ k_{\theta H} \end{bmatrix}$$

$$= \left\{ \begin{array}{c} \omega^2 \left(m_1 H_1 \frac{\ddot{U}_1}{\ddot{U}_{eff}} + (I_0 + I_1) \frac{\ddot{\theta}_0}{\ddot{U}_{eff}} \right) \\ \omega^2 \left(m_0 \frac{\ddot{U}_0}{\ddot{U}_{eff}} + m_1 \frac{\ddot{U}_1}{\ddot{U}_{eff}} \right) \\ \cdot \\ \cdot \end{array} \right\} \tag{15}$$

which can be rewritten as,

$$[A] \left\{ \begin{array}{c} k_{HH} \\ k_{H\theta} \\ k_{\theta H} \\ k_{\theta\theta} \end{array} \right\} = \{f\} . \tag{16}$$

Matrix **A** is rectangular and contains twice as many rows as the number of time-domain windows in which equilibrium is being imposed. Using a generalized inverse of matrix **A**, a solution of this equation can be obtained as,

$$\left\{ \begin{array}{c} k_{HH} \\ k_{H\theta} \\ k_{\theta H} \\ k_{\theta\theta} \end{array} \right\} = ([A^*]\,[A])^{-1}\, \{f\} , \tag{17}$$

where $\mathbf{A}^*$ denotes the hermitian transpose of matrix $\mathbf{A}$. Since the ratios of the Fourier amplitude is construed in this paper to signify the transfer function between the respective quantities, the above equations cannot be evaluated at single frequencies. Instead, a sliding window is used to perform over the data, within which the transfer functions are evaluated resulting in a single row in equation (16). This sliding window is characterized by its width and its pitch. The traditional frequency-based transfer function between the free field motion and the motion as measured at the pile cap, $\frac{\ddot{U}_0}{\ddot{U}_{eff}}$, is shown in figure (5). Figures (6) and (7) show the transfer functions between the free field motion and the motion as measured at the pile cap, $\frac{\ddot{U}_0}{\ddot{U}_{eff}}$, evaluated in the time-frequency domain, by segmenting the data records as described above. These figures correspond, respectively, to a window size of length 800 and 2000 observations, respectively. It is noted that the maximum amplification of the free field motion is quite sensitive to the breakdown in time of the frequency content of the vibrations. Specifically, a smaller amplification is felt by the structure as a result of that. This is as expected, since the total energy being delivered to the structure is being spread over a finite time interval. The reduction in the amplification is, of course, dependent on the length and pitch of the window used in the probing the data. Figure (8) shows the impedance function calculated according to the least squares procedure described above. Only the translational components of impedance matrices are shown. The figure shows results corresponding to three analyzing windows containing, respectively, 400, 800, and 2000 observations each, and each plot shows results from both the 89 and the 268 Gal experiments. This is consistent with the results regarding the transfer function and suggests that the soil medium can be effectively stiffer than is suggested by a traditional Fourier analysis. It is clear that the choice of the window size is critical for the proper evaluation of the impedance functions. It is believed that the appropriate window size should be related to the relaxation time of the system. Additional work seems to be required on this topic.

CONCLUSIONS

This paper presents results from the analysis of experimental data related to the dynamic behavior of pile groups. By idealizing the structure-soil system as a three degree of freedom model, equations are obtained and implemented for the impedance function of the soil-pile system. A least squares procedure is developed for estimating the impedance functions of the soil-pile system. The procedure relies on frequency dependence of the impedance functions being time-invariant. The correctness of this postulate is investigated through a joint time-frequency analysis of various transfer functions involved in the evaluation of the impedance functions. It is found that these transfer functions are somewhat sensitive to the length of the analyzing window in a moving-window Fourier analysis. It is found the peaks of transfer functions, as well as, their frequency spread is dependent on the length of this window, in general resulting with wider spread and smaller amplification with the smaller windows. This suggests that the parameters of the superstructure should be taken into consideration in deciding on an appropriate window size.

The time-frequency characteristics of the system notwithstanding, valuable information regarding the nonlinear behavior of the soil and its interaction with the pile-structure system is obtained. In particular, it is observed that the nonlinear behavior of the soil is restricted to the topmost layer of the soil, and that it diminishes with depth. It is also observed that under strong ground shaking, the interaction of the soil with the pile system is dminished as the soil tends to vibrate at its own dominant period, which in that case, however, is associated with a significant damping level. A strong dispersive behavior of the soil is also observed, which is reflected by a dependence of the shear wave velocity on frequency.

ACKNOWLEDGMENTS

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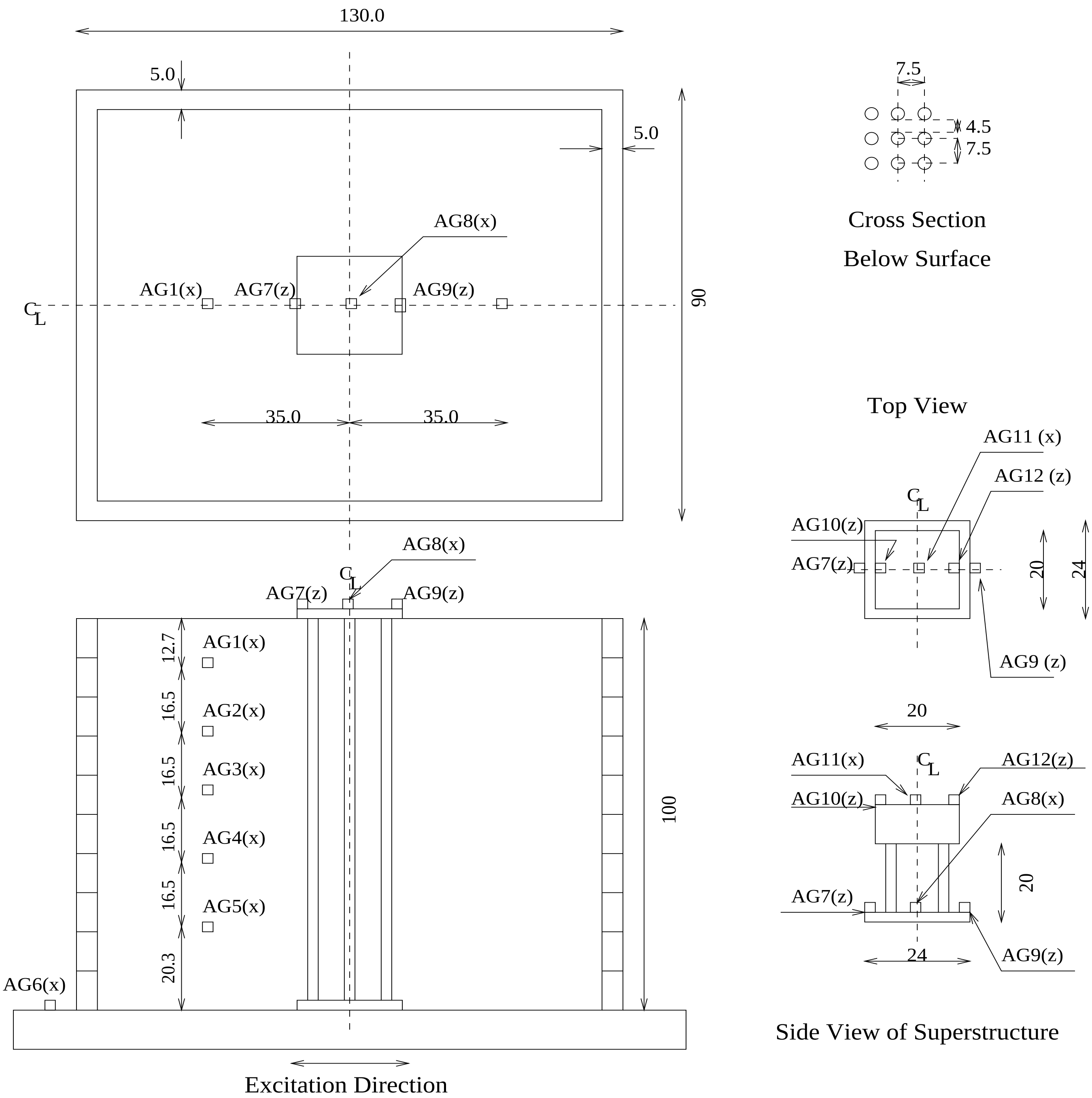

Figure 1: Experimental Set-Up Showing Dimensions and Accelerometer Locations; All Dimensions are in cm.

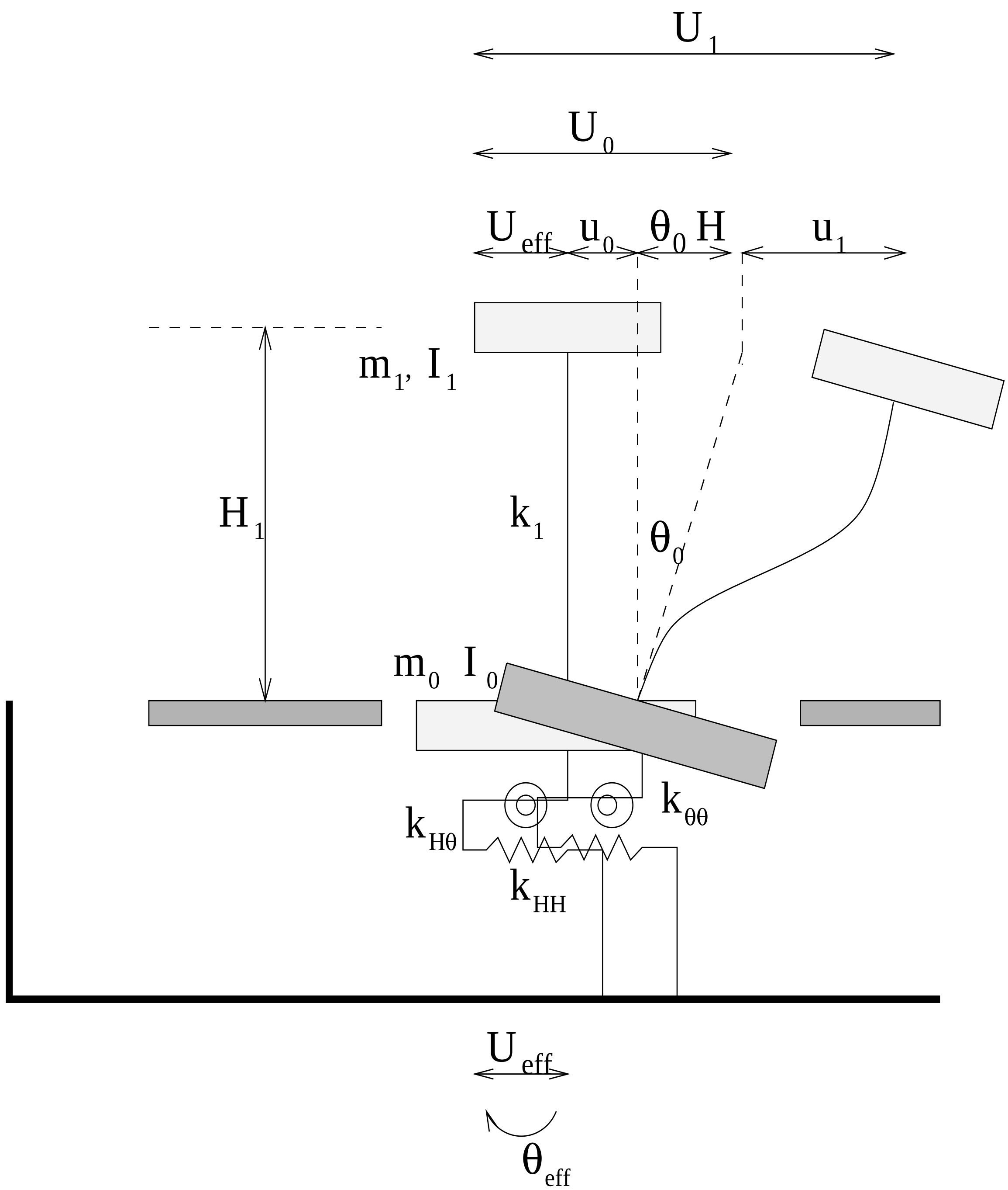

Figure 2: Super-Structure System with Pile Head Impedances.

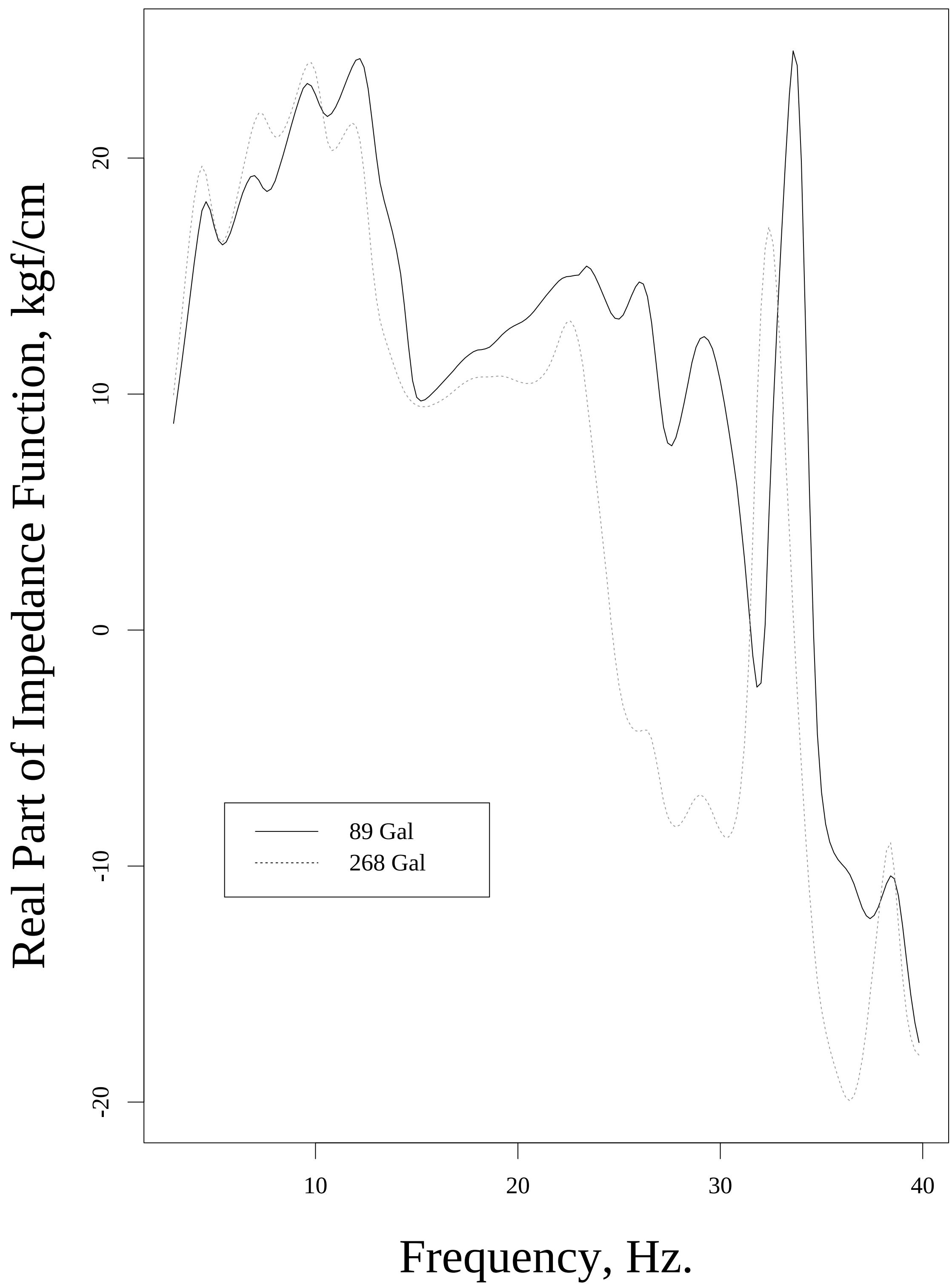

Figure 3: Real Part of the Impedance Function; Acceleration Records.

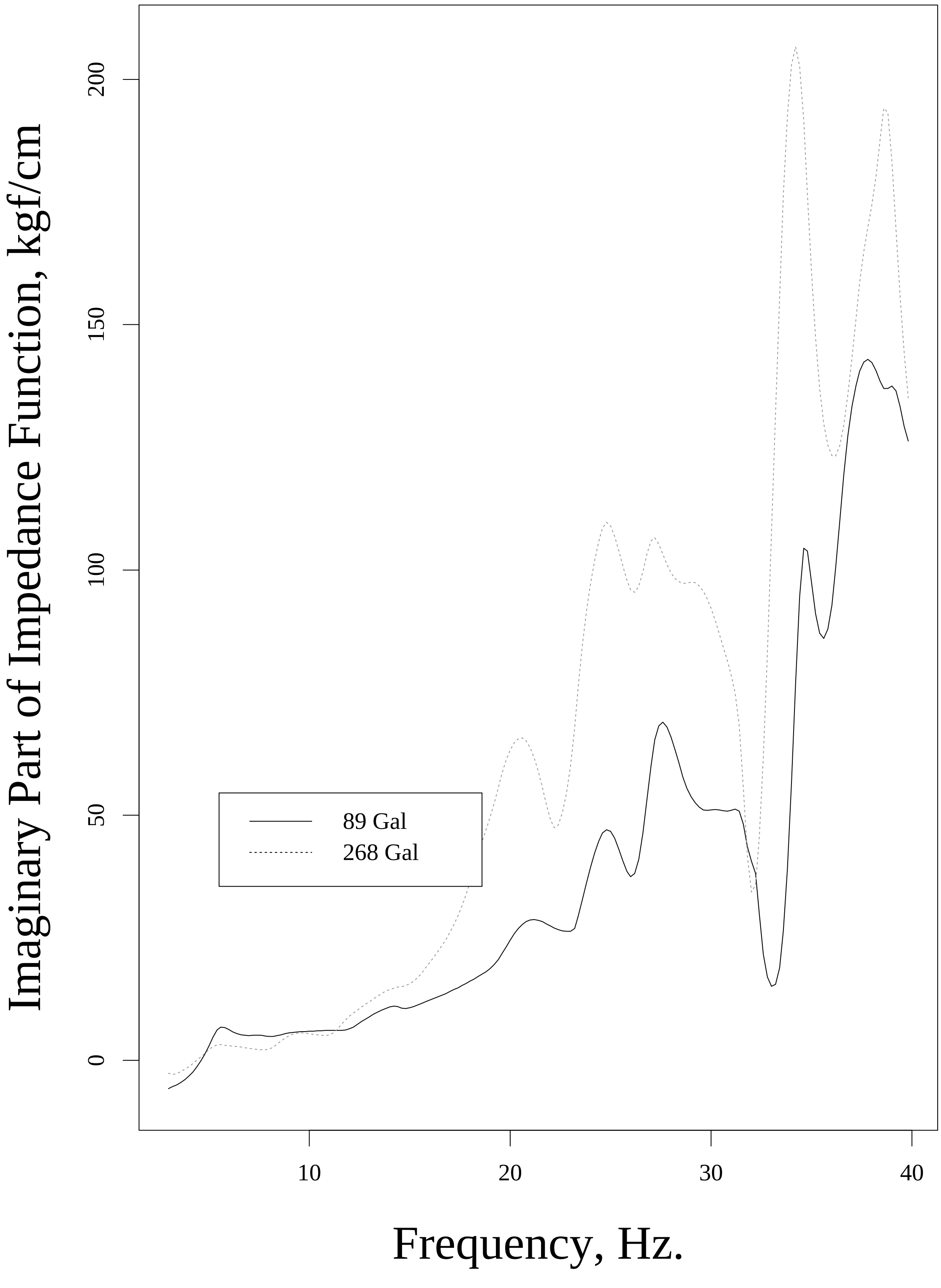

Figure 4: Imaginary Part of the Impedance Function; Displacement and Acceleration Records.

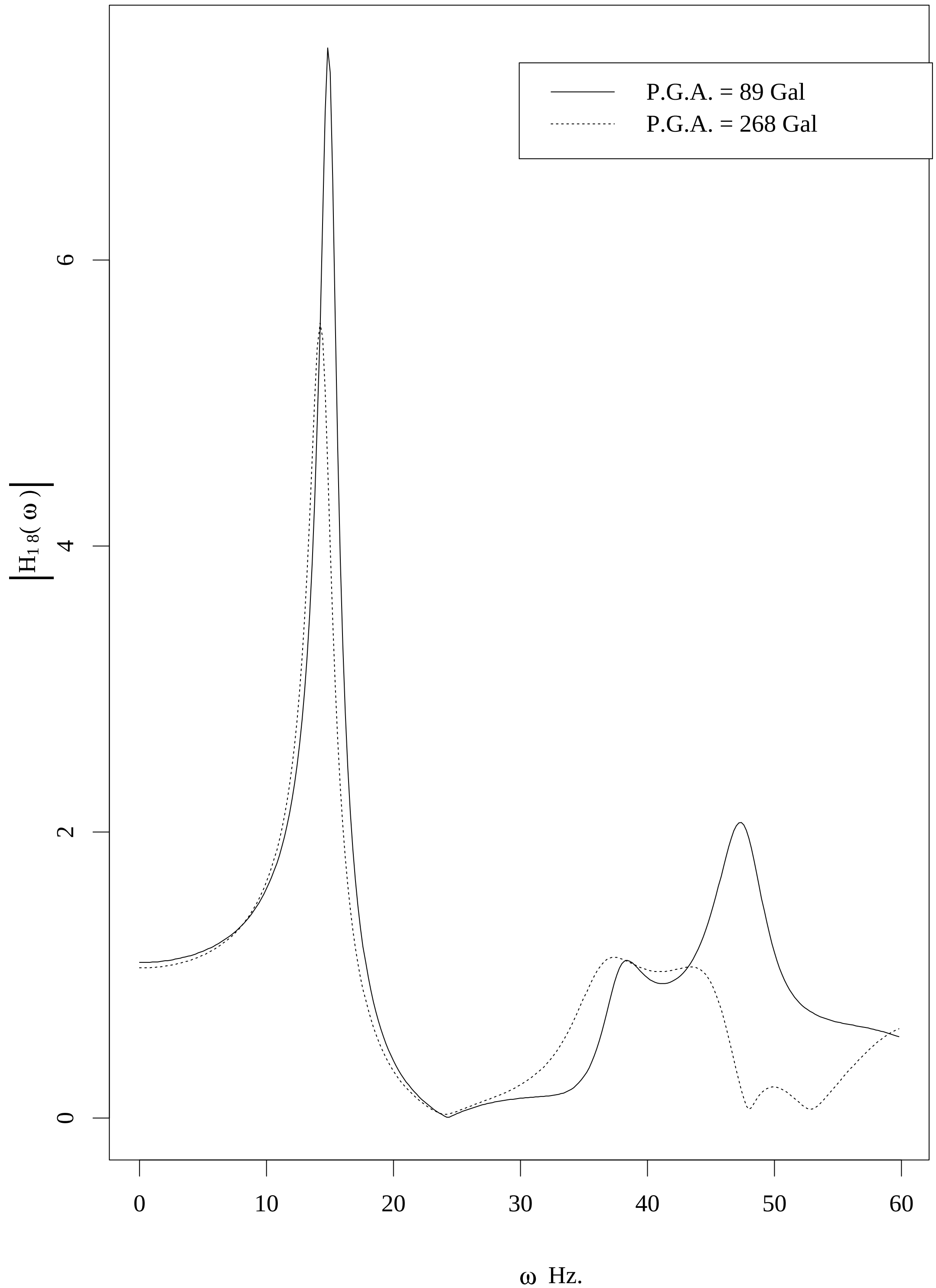

Figure 5: Frequency Plot of the Transfer Function between the Shaking Table and the Pile Cap.

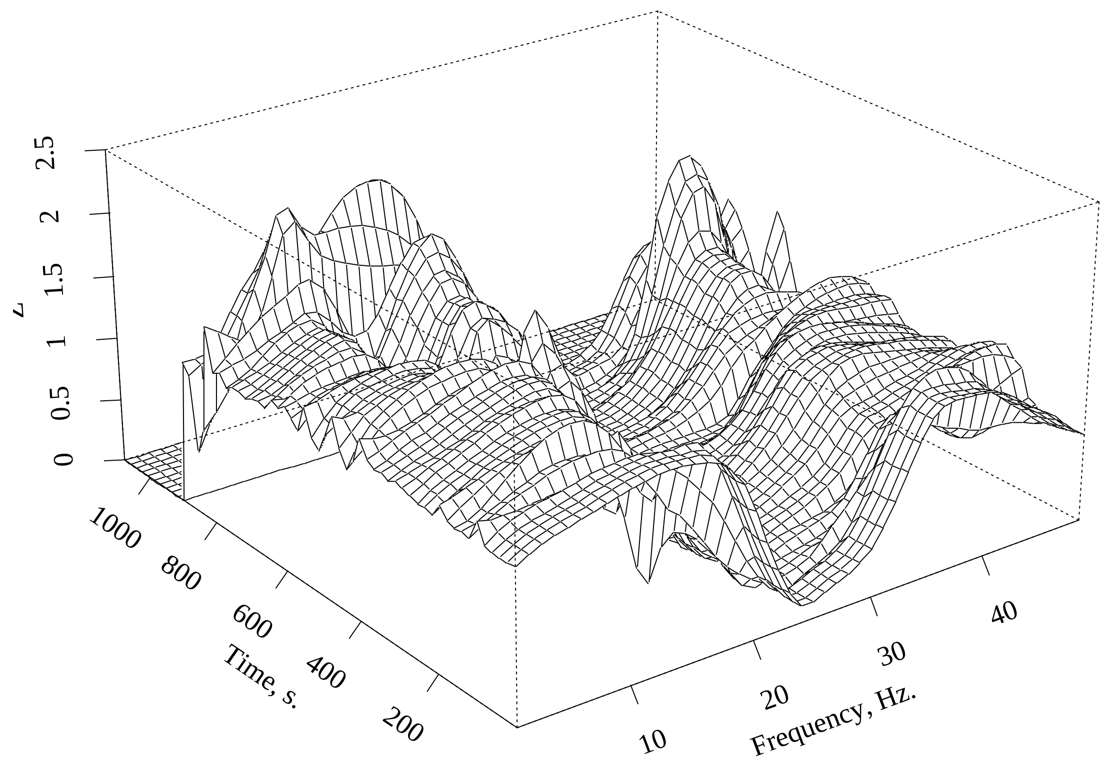

Figure 6: Time-Frequency Plot of the Transfer Function between the Shaking Table and the Pile Cap; Peak Ground Acceleration = 268 Gal; 800 Observations in Sliding Window.

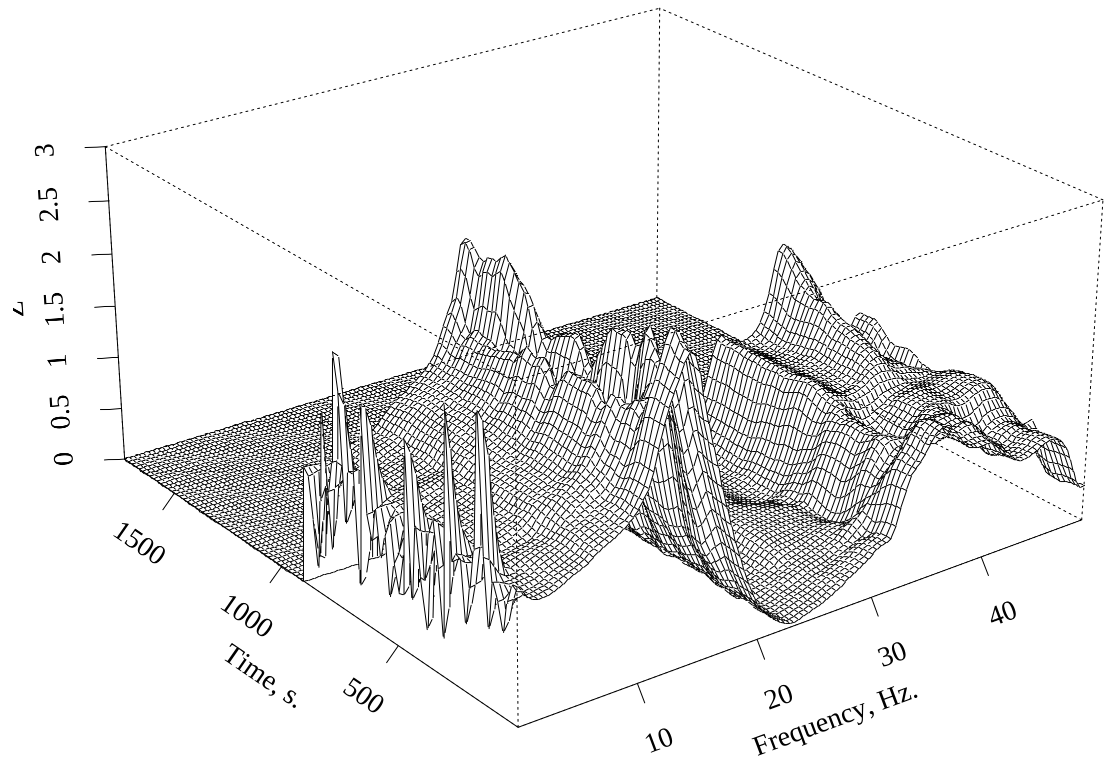

Figure 7: Time-Frequency Plot of the Transfer Function between the Shaking Table and the Pile Cap; Peak Ground Acceleration = 268 Gal; 2000 Observations in Sliding Window.

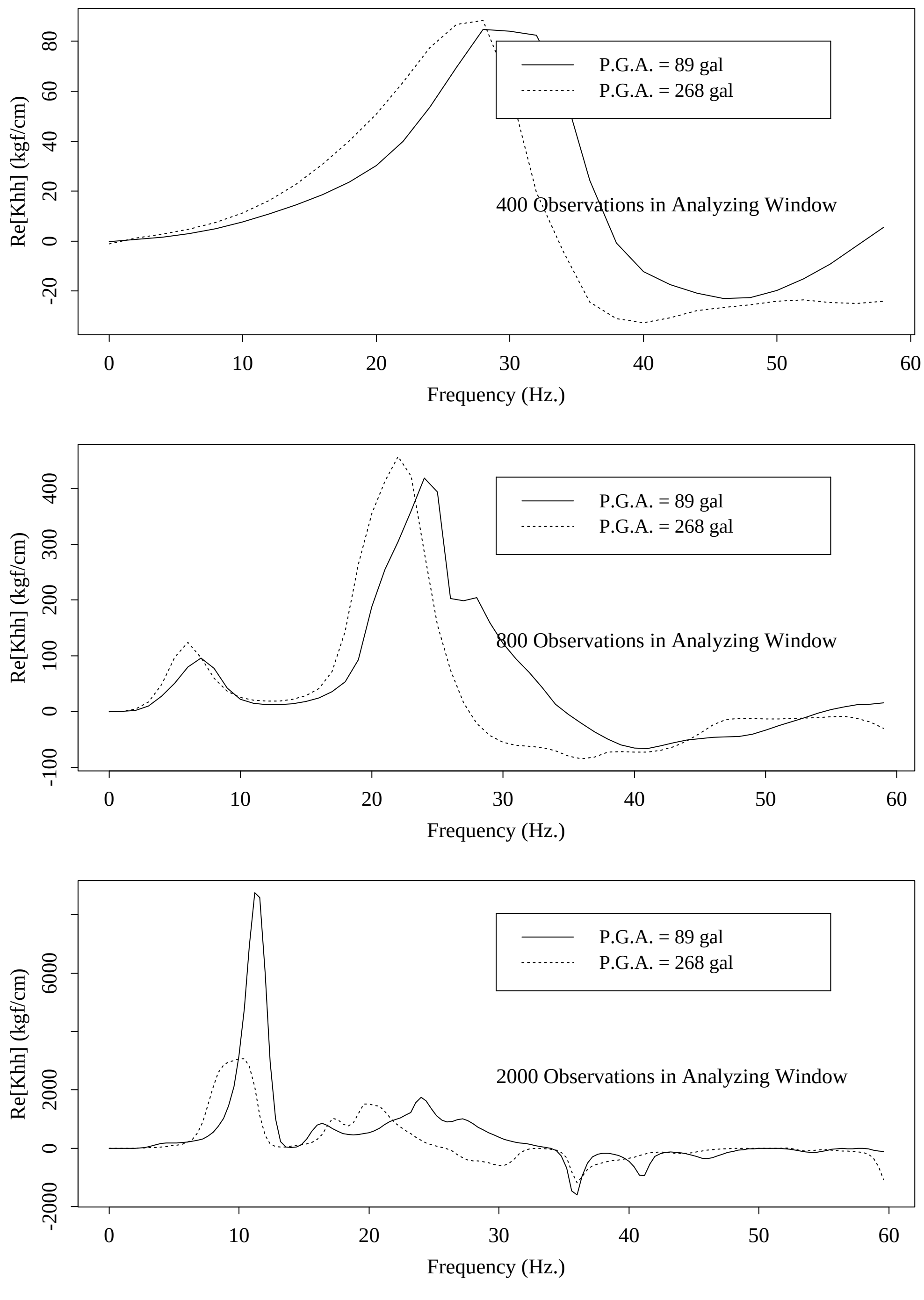

Figure 8: Real Part of the Translational Impedance Function of the Pile Cap; Peak Ground Acceleration = 268 Gal; Window Size = 400 Observations; Number of Samples = 50.

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