AN ANALYSIS FOR STRESS DISTRIBUTION OF PILED RAFT FOUNDATIONS UNDER SEISMIC LOADING

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ABSTRACT

This paper describes a study on the behavior of piled raft foundations under dynamic/static horizontal loading. The finite element analysis was used for the dynamic loading. It was found from the analysis that: the change of the impedance function due to adding piles to the raft is relatively small but the foundation input motion is affected significantly, the contribution of the raft accounts for 60 to 80 per cent of the load bearing ratio. Based on this result, a simplified method of static analysis which is intended for use in the practical design has been proposed. In the analysis, three different interactions, i.e. raft to pile, pile to raft and pile to pile interactions, are separately taken into consideration. The examination has indicated that the method can give results which compare to those obtained from the dynamic analysis.

Keywords: Piled Raft Foundation, Soil-Structure Interaction, Finite Element Analysis, Approximate Analysis, Design

1 INTRODUCTION

The piled foundation is normally used when constructing buildings on soft soils. The spread foundation, however, becomes an alternative when appropriate load bearing soil layers do not exist. In the latter case, from the viewpoint that the excessive settlement and differential settlement have to be avoided, the use of a composite foundation is becoming very popular in recent years. This composite foundation consists of a spread foundation, usually a raft foundation, and a relatively few number of friction piles and is called a piled raft foundation. In the case of the piled raft foundation, the load bearing mechanism is fairly complex because the load is transmitted to the ground through the raft and the piles.

The vertical load bearing mechanism has been extensively investigated by a number of researchers by applying the elasticity theory [Poulos, 1994; Randolph, 1994] and the finite element method [Yamashita, 1998]. Based on these results, piled raft foundations are becoming popular in practical use [Yamada, 1998].

The load bearing mechanism under horizontal loading or during earthquakes, however, has not been studied in detail. This is partially because piled raft foundations are considered as raft foundations in the current design practice. Since the behavior of the piled raft foundation during earthquakes is considered fairly complex due to the dynamic interaction among the raft, the piles and the ground, the design procedure should include the effect of this mechanism in a certain manner.

The objective of this paper is to investigate the possibility of the design method of the piled raft foundation against earthquakes. An extensive study of the piled raft behavior during dynamic loading is first made and based on its results a simplistic elasticity based procedure is then proposed.

2 DYNAMIC BEHAVIOR OF PILED RAFT FOUNDATION

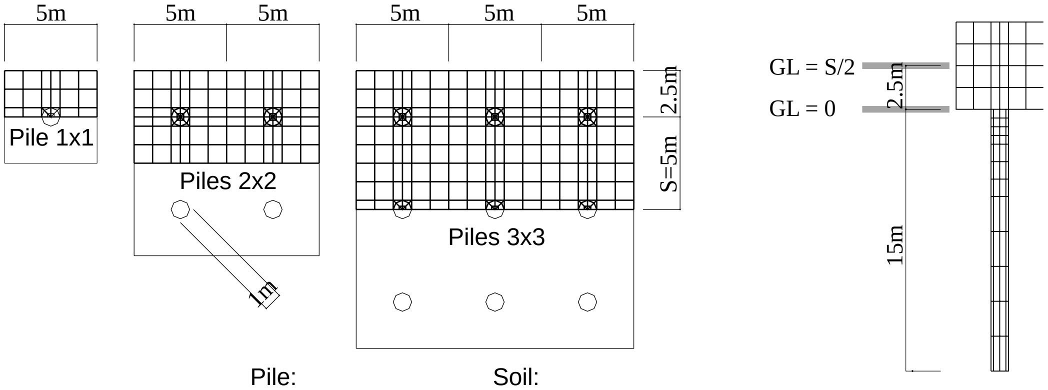
The dynamic behavior of the piled raft foundation placed on the uniform elastic soil is studied based on the three dimensional finite element analysis.

2.1 Method of Analysis

In the analysis, the raft is considered as a massless rigid body, the rest of the system, i.e., the pile-soil system, is assumed as a linear elastic body. The three dimensional finite element analysis based on the dynamic substructure method has been used to compute impedance functions, foundation input motions, load bearing ratios and stress distributions along the pile. A computer code ACS SASSI was used and the analysis was made in the frequency domain.

2.2 Analysis Model

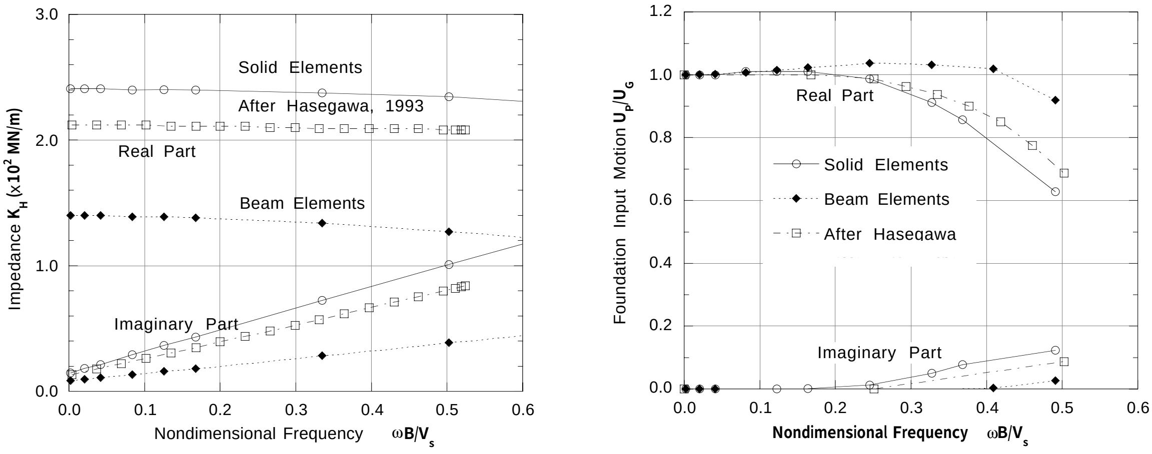
Dynamic properties and the finite element mesh layout used in the analysis are shown in Figure 1. The diameter of



Pile:
E_p = 20594MN/m^2
ν_p = 0.167
ρ_p = 2.4t/m^3
h = 0.03

Soil:
V_s = 150m/s
ν_s = 0.45
ρ_s = 1.5t/m^3
h = 0.03

Figure 1 Analysis Model (Finite Element Mesh Layout)



(a) Horizontal Impedance Function

(b) Foundation Input Motion

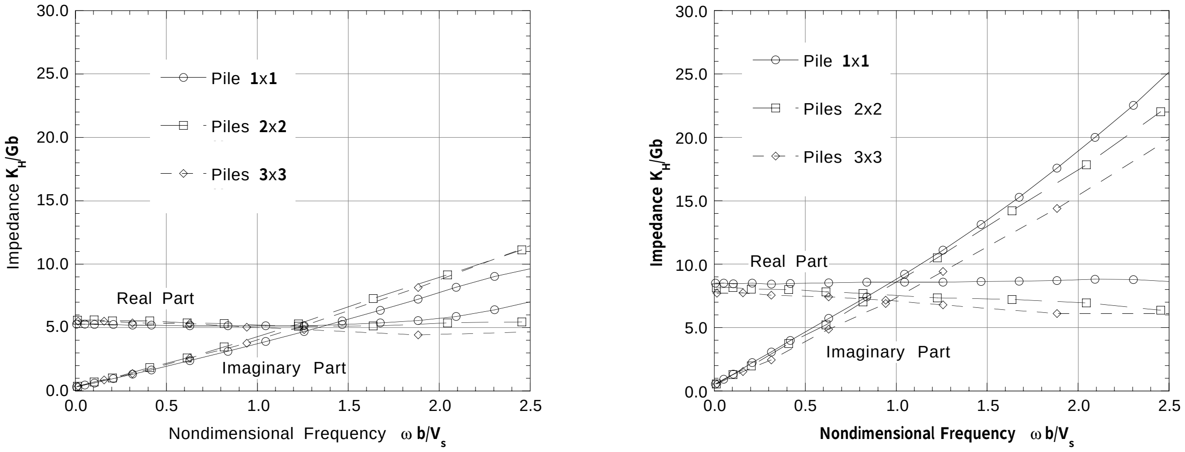
Figure 2 Comparison between Solid Elements and Beam Elements

the pile is 1 m and the length of the pile is 15 m. The raft is square and has 1, 4 (2 x 2) or 9 (3 x 3) piles, in which the distance between piles is set to 5 m The embedment of the raft foundation has been considered in addition to the non-embedded case.

Piles are often modeled as beams in the finite element analysis due to its flexural characteristics. However, since beams do not occupy any volumes in the three dimensional space, the direct use of a beam as a pile in conjunction with solid elements as soils is not appropriate in the dynamic soil-structure interaction analysis. The reason is because a pile modeled as a beam has very small diameter hence it tends to have small resistance. Figure 2 shows impedance functions and foundation input motions of single piles. It is confirmed by this result that the beam element modeling underestimates the impedance function and overestimates the foundation input motion. According to this study, piles are modeled by solid elements in this paper, as shown in Figure 1.

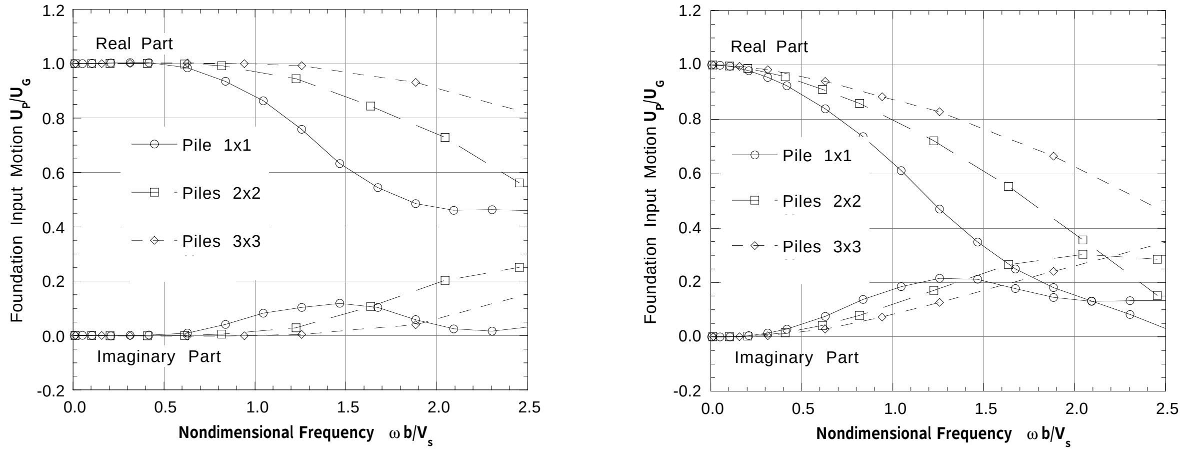
2.3 Impedance Functions

Figure 3 shows the impedance function K_H of the piled raft foundation. In the figure, G represents the shear modulus of the soil, b the half-width of the raft, and V_s the shear wave velocity of the soil.



(a) Raft with No Embedment (b) Raft with Embedment

Figure 3 Horizontal Impedance Function



(a) Raft with No Embedment (b) Raft with Embedment

Figure 4 Foundation Input Motion

From this figure, it is seen that the impedance increases very little with the increase of the number of piles, indicating the small contribution of the piles to the impedance function. In the case of embedded foundations, the normalized impedance function decreases with the increase of the number of piles. This may be resulted from the fact that the embedment is kept constant while increasing the size of the raft and the number of piles. However, the difference is relatively small.

2.4 Foundation Input Motions

Figure 4 shows the foundation input motion which is computed as the response, U_P, of the raft foundation at the ground surface level due to the vertically incident S wave with respect to the response, U_G, of the free field also at the ground surface level.

From this figure, it is observed that the real part of the foundation input motion decreases with increasing frequency while the imaginary part increases as the frequency increases. This tendency becomes more apparent when the foundation has embedment.

2.5 Load Bearing Ratio

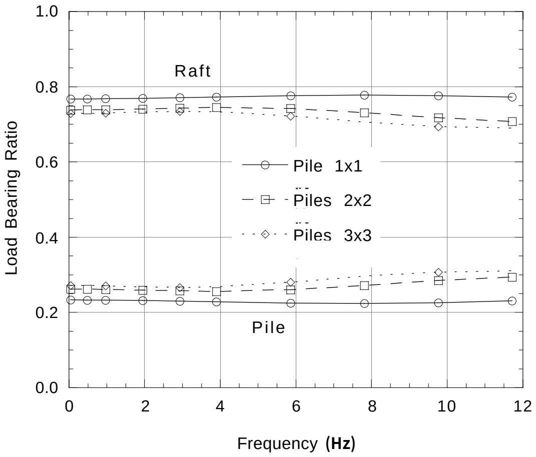
Figure 5 shows the load bearing ratio, i.e., the ratio between the resistance of the raft and that of the piles, each of which was computed by integrating the element stresses in the horizontal direction.

From this figure, it is found that the raft bears most of the load and the frequency has little influence on the ratio. This tendency is the same between two cases, impedance analysis and incident wave analysis.

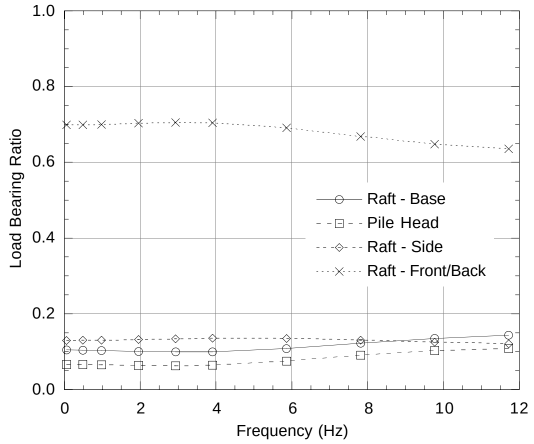
2.6 Stress Distribution of Pile

Figure 6 depicts the distribution of the bending moment of the pile. The bending moment of the pile was obtained by placing very soft beam elements and extracting the resulting slope of these elements.

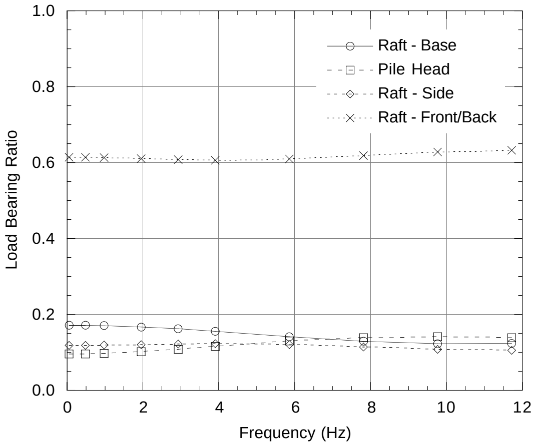
From this figure, it is observed that the bending moment is large at the upper part of the pile near the ground/the base of the foundation, especially when the frequency is low. This tendency becomes vague in the case of the incident wave, but the value is still large at the upper part of the pile.



(a) Raft with No Embedment, Forced Vibration



(b) Raft with Embedment, 2x2 Piles, Forced Vibration



(c) Raft with Embedment, 2x2 Piles, Incident Wave

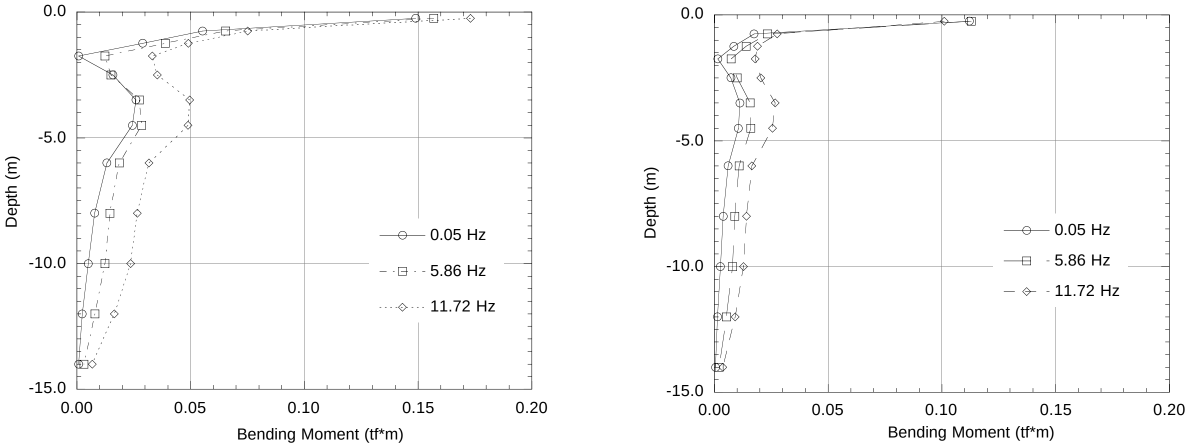
Figure 5 Load Bearing Ratio

3 APPROXIMATE ANALYSIS OF PILED RAFT FOUNDATION

3.1 Method of Analysis

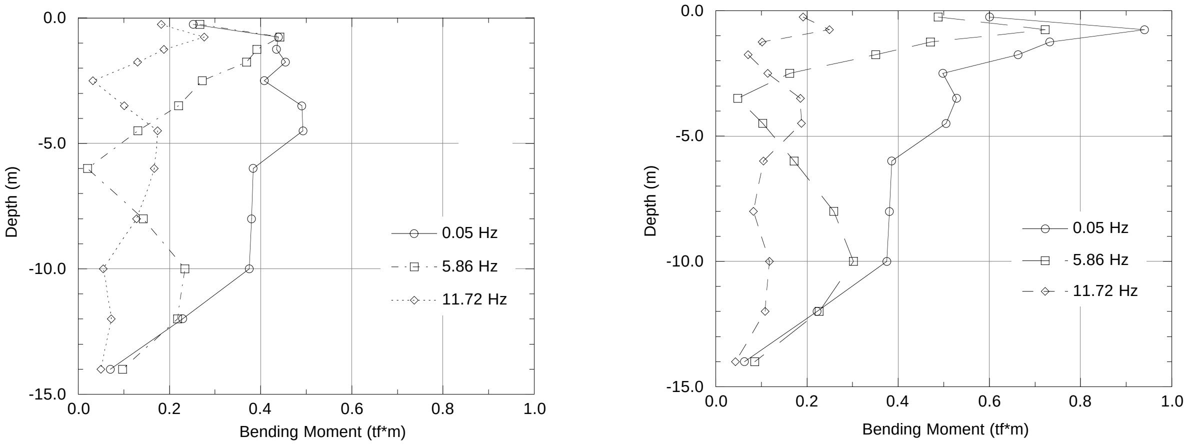
It has been pointed out from three dimensional finite element analysis that the load bearing ratio is almost frequency independent and that the bending moment of the pile is large near the ground surface/the base of the raft which is quite similar to the static case. Based on this, an approximate method of analysis of the piled raft foundation under static lateral loading is proposed [Mano and Nakai, 2000].

In the proposed method, the raft-pile-soil interaction system is divided into two: a raft foundation on the surface of the ground and piles installed in the ground, as shown in Figure 7. Thus, the following interactions are considered in the analysis:
(1) influence of the friction at the base of the raft on the piles,
(2) influence of the piles under lateral loading on the raft, and
(3) influence of the piles under lateral loading on other piles.
Secondary effects other than these are neglected in the analysis.



(a) 2x2 Piles / Raft with No Embedment / Forced Vibration

(b) 2x2 Piles / Raft with Embedment / Forced Vibration



(c) 2x2 Piles / Raft with No Embedment / Incident Wave

(d) 2x2 Piles / Raft with Embedment / Incident Wave

Figure 6 Distribution of Bending Moment of the Pile

3.1.1 Raft to Pile Interaction

The deflection due to the friction at the base of the raft can be obtained by

a) computing the displacement in the ground along the pile shaft (note: piles do not exist) due to the friction at the base of the raft placed on the ground surface, and then

b) computing the deflection of the pile due to this ground movement.

In order for this, the distribution of the friction at the base of the raft should be determined in the first place. This can be done by dividing the raft foundation into a number of small elements and applying the same displacement to each element, as shown in Figure 8. Resulting reaction forces of the elements represent the distribution of the friction. Influence factors between arbitrary two elements can be obtained by applying the theory of elasticity [Mogami, 1977]:

$$\delta_{ji} = I_{r\theta} q_i$$

$$I_{r\theta} = -\frac{a(1+\nu_s)}{\pi E_s}\left\{(A+B)\cos^2\theta + (A-B)\sin^2\theta\right\}$$

$$A = \frac{\nu_s a^2}{3r^2}\left\{\left(2-\frac{r^2}{a^2}\right)\left(1+\frac{r}{a}\right)E(k) - \left(1-\frac{r}{a}\right)\left(2+\frac{r^2}{a^2}\right)K(k)\right\}$$

$$B = (2-\nu_s)\left\{\left(1+\frac{r}{a}\right)E(k) + \left(1-\frac{r}{a}\right)K(k)\right\} \tag{1}$$

$$k^2 = \frac{4r/a}{(1+r/a)^2}$$

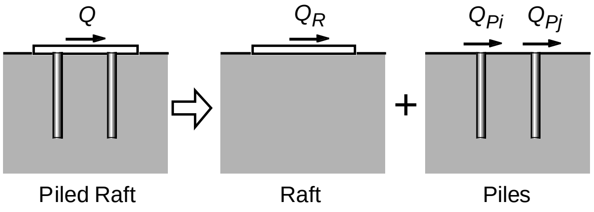


Figure 7 Decomposition of Piled Raft Foundation under Horizontal Loading

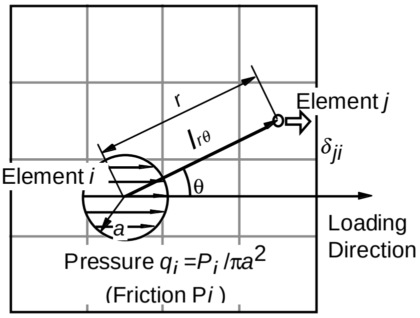


Figure 8 Subdividing Raft into Elements

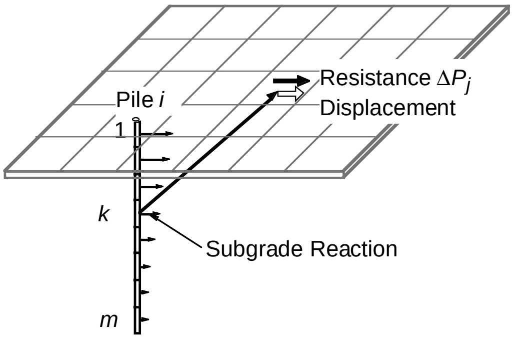


Figure 9 Pile to Raft Interaction

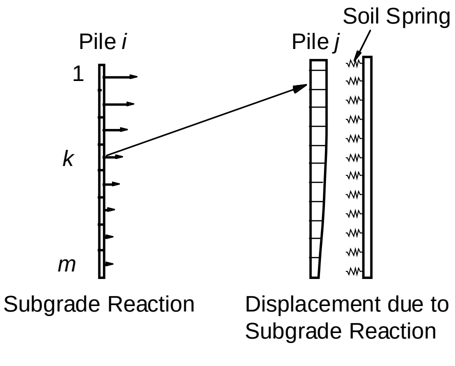


Figure 10 Pile to Pile Interaction

where K and E are the complete elliptic integrals of the first and second kind, respectively. The displacement of Element j, δ_j, can be obtained by summing up the contributions, δ_{ji}, from other elements. Based on this, the displacement of the raft, $\delta_{R,R}$, due to a unit lateral load is computed.

Once the distribution of the friction force is determined, the displacement along the pile shaft in the ground due to this force can then be computed again by the theory of elasticity. The horizontal displacement u at an arbitrary point (r, θ, z) in the ground due to a distributed horizontal load q_0 in a circular area of the radius a on the surface of the ground is given by [Mogami, 1977],

$$
\begin{aligned}
u &= \frac{q_0 a}{4G}\left\{(C+D)\cos^2\theta + (C-D)\sin^2\theta\right\} \\
C &= -\int_0^\infty (2\nu + \xi z)e^{-\xi z}\frac{1}{\xi}J_1(\xi a)J_2(\xi r)d\xi \qquad (2) \\
D &= \int_0^\infty \{\xi z - 2(2-\nu)\}e^{-\xi z}\frac{1}{\xi}J_1(\xi a)J_0(\xi r)d\xi
\end{aligned}
$$

where, G is the shear modulus of the ground, ν is the Poisson's ratio of the ground and J_n is a Bessel function of the n-th order.

The deflection of the pile due to this ground movement can be computed based on the so-called Winkler model, i.e. a beam on elastic subgrade [Kishida and Nakai, 1977].

3.1.2 Pile to Raft Interaction

In this step, the displacement of the raft and the friction force of the raft due to a unit horizontal load acting at a pile head is computed. This can be done by:

a) computing the distribution of the subgrade reaction along the pile based on the Winkler hypothesis, and
b) computing the displacement of the raft, $\delta_{R,Pi}$, or the friction force of the raft due to this distributed load by subdividing the raft into small elements and applying the theory of elasticity.

The schematic illustration of the procedure is shown in Figure 9.

3.1.3 Pile to Pile Interaction

This considers the interaction among piles. As in 3.1.2, the procedure is the following:

a) compute the distribution of the subgrade reaction along the pile due to a unit horizontal load at its head based on the Winkler hypothesis, and
b) compute the deflection of the pile, $\delta_{Pj,Pi}$, due to this displacement again based on the Winkler model.

The procedure is illustrated in Figure 10.

3.1.4 Load Bearing Ratio

The load bearing ratio between the raft and the piles can be obtained by combining previously developed relationships. Suppose the bearing load of the raft is Q_R and that of each pile is Q_{Pi} (i=1 ~ n), then the displacement u_R of the raft and the deflection u_{Pi} at the head of each pile can be expressed by the following equations:

$$
\begin{aligned}
u_R &= \sum_{i=1}^{n}\delta_{R,Pi}Q_{Pi} + \delta_{R,R}Q_R \\
&\qquad\qquad\qquad\qquad (3) \\
u_{Pi} &= \sum_{j=1}^{n}\delta_{Pi,Pj}Q_{Pi} + \delta_{Pi,R}Q_R
\end{aligned}
$$

Considering additional compatibility and equilibrium conditions:

$$
\begin{aligned}
&u_R = u_{P1} = u_{P2} = \cdots = u_{Pn} \\
&\sum_{i=1}^{n}Q_{Pi} + Q_R = Q \quad (Q\text{: Total Horizontal Load}) \qquad (4)
\end{aligned}
$$

it is possible to come up with the two bearing loads, hence the load bearing ratio between the raft and the piles.

3.2 Deflection and Bending Moment of Piles

Since the computational procedure of the proposed method consists of the three interaction phases, it is possible to examine the effect of each interaction on the behavior of the piled raft foundation.

3.2.1 Effect of Interaction on Pile Deflection and Bending Moment

In order for this, a piled raft foundation with 4 (2x2) piles, as shown in Figure 11, has been examined. Figure 12 shows the deflection and the bending moment of the pile due to:

(A) the friction force at the base of the raft,
(B) the bearing load at the head of the pile, and
(C) the deflection of other piles through the ground.

From this figure, it is found that contributions due to (A), (B) and (C) are 55%, 30% and 15%, respectively, for the deflection at the pile head. The percentage of (A) for the bending moment is a little smaller than this value. However, this fact indicates that the friction force has a fairly large influence even at the pile head. For the embedded part located deep in the ground, the effect of the bearing load at the pile head is small as is expected.

3.2.2 Load Bearing Ratio

Figure 13 shows the load bearing ratio for the piled raft foundations with 1, 4 (2 x 2) or 9 (3 x 3) piles. The properties of the soil and the piles are the same as those shown in Figure 11. From this figure, it is seen that the load bearing ratio of the raft goes down gradually with increasing the number of piles. The result compares to that of the dynamic analysis which is shown in Figure 5. It is also pointed out from Figure 13 that the contribution of the corner piles is twice as large as that of the center pile, indicating the group pile effect.

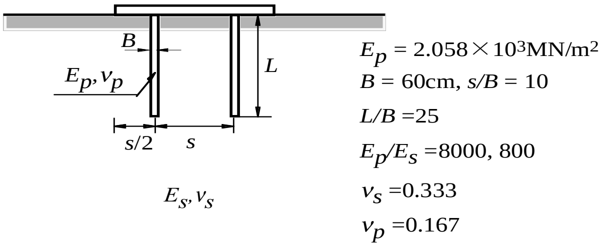
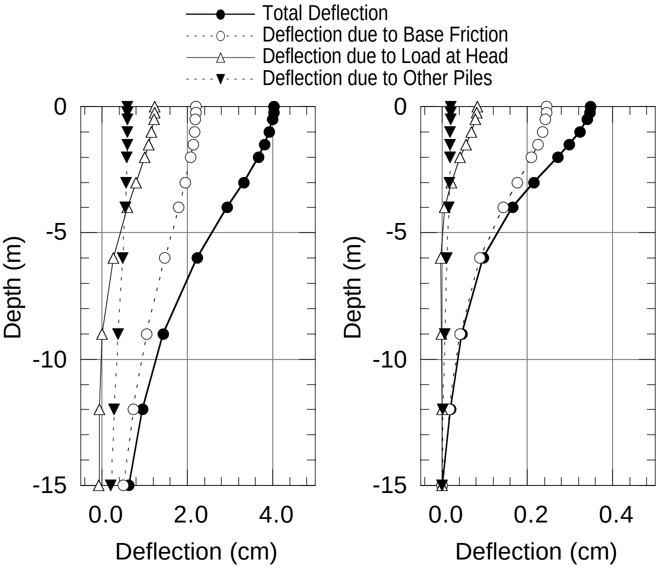
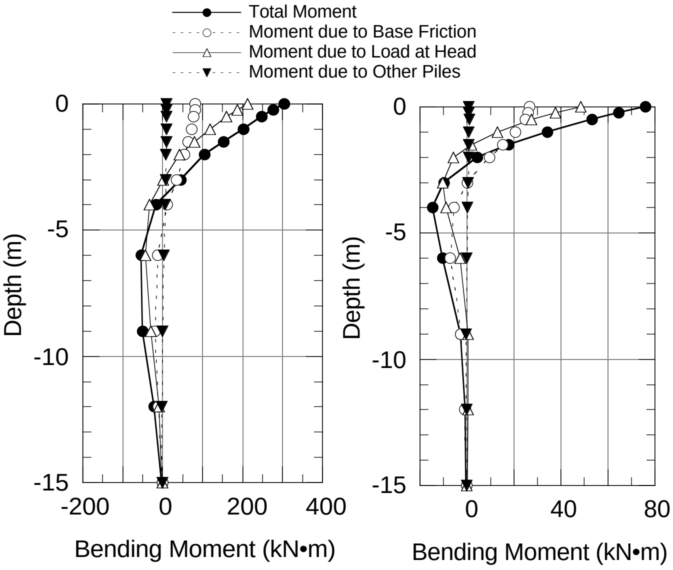


Figure 11 Dynamic Properties of Analysis Model



(a) Ep/Es=8000 (b) Ep/Es=800



(c) Ep/Es=8000 (d) Ep/Es=800

Figure 12 Deflection and Bending Moment due to Interaction

4 CONCLUSIONS

In this paper, the behavior of the piled raft foundation under dynamic/static horizontal loading has been examined. It was found from the analysis that

(1) Solid elements rather than beam elements are preferable for modeling piles in the finite element analysis.
(2) The impedance function of the piled raft foundation increases as the number of piles increases. The extent to which it increases is relatively small due to the smaller number of piles.
(3) As for the foundation input motion, the real part decreases and the imaginary part increases as the frequency increases.
(4) The contribution of the raft accounts for 60 to 80 per cent of the load bearing ratio. The percentage is even higher when the foundation has embedment.

Based on this result, a simplified method of static analysis which is intended for use in the practical design has been proposed. The examination has indicated that the method can give results which compare to those obtained from the dynamic analysis. It is also pointed out that three different interactions, i.e. raft to pile, pile to raft and pile to pile interactions, can be examined separately in detail.

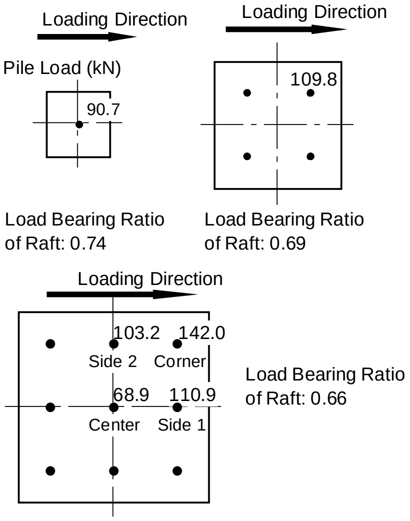


Figure 13 Load Bearing Ratio

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