RESPONSE SPECTRUM METHOD FOR EVALUATING NONLINEAR AMPLIFICATION OF SURFACE STRATA

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ABSTRACT

This paper proposes a response spectrum method (RSM) for evaluating nonlinear amplification of surface strata (SS) overlying an engineering bedrock (EBR). The proposed RSM follows the procedures below. (1) Represent the response characteristics of SS on EBR by a multi-lumped mass model with shear springs and damping coefficients. (2) Calculate a period T_1, a mode shape $\{U_1\}$ and a modal damping factor ζ_1 for the first vibration mode. (3) Evaluate the equivalent shear wave velocity Vs_e of SS. (4) Evaluate the amplification factor R_1 of the surface to EBR at T_1 using the Haskell matrix of two strata consisting of an equivalent SS with Vs_e and ζ_1 on EBR. (5) Evaluate the displacements in SS from $R_1\{U_1\}$, and hence evaluate the shear strains. (6) In accordance with nonlinear relations between shear modulus (G), damping factors (h) and shear strains, replace soil constants G and h by those corresponding to strains. (7) Return to step (1) with revised G and h. Iterate these procedures until T_1 becomes stable. (8) After T_1 is converged, calculate the response distributions in SS. The results obtained by the proposed RSM are compared with those by "SHAKE" in terms of the response spectrum at the surface as well as distributions of displacement, shear wave velocity and damping factor in SS. The applicability of the proposed RSM for evaluating the nonlinear amplification of SS is confirmed through the comparative studies.

KEYWORDS

soil response; response spectrum method; nonlinearity of subsoil; nonlinear amplification characteristics; modal analysis; first natural period; first mode shape; shear wave velocity; damping factor; response displacement

1. INTRODUCTION

Both the non-linearity of the surface strata (SS) and the impedance ratio between SS and the underlying engineering bedrock (EBR) affect the earthquake response of SS during a severe earthquake. When a design earthquake input motion is given at an outcrop of EBR in the form of a time history, the nonlinear response of SS can be calculated by means of a time history nonlinear analysis method (THNA) such as a computer code "SHAKE". However, when the design earthquake input motion is prescribed only in the form of a response spectrum instead of a time history, THNA can no longer be employed. Although a conventional response spectrum method (RSM) is practical and effective for an anti-seismic design analysis of buildings, this method is not directly employed for the nonlinear response analysis of SS on EBR because the amplification due to the impedance ratio can not be incorporated into it.

This paper proposes a RSM for evaluating the nonlinear amplification of SS. Unlike the conventional RSM, the proposed RSM can evaluate the nonlinear amplification of SS due to the non-linearity of SS and the impedance ratio between SS and EBR. Furthermore, comparative studies with a more rigorous method "SHAKE" are performed to verify the applicability of the proposed method.

2. ANALYSIS METHOD AND ANALYSIS MODEL

The proposed response spectrum method follows the procedures below.

1) Transformation of acceleration response spectra of input motion

Acceleration response spectra $S_A(T, \zeta)$ at a period T and a damping factor ζ are estimated from the acceleration response spectrum $S_{AT}(T, \zeta=0.05)$ which is prescribed as an outcrop motion at an engineering bedrock. The relation between acceleration response spectrum $S_A(T, \zeta=0)$, velocity response spectrum $S_V(T, \zeta=0)$ and an acceleration Fourier spectrum $F_A(T)$ is given by Eq. (1).

$$F_A(T) \approx S_V(T, \zeta=0) \approx (T/2\pi)\, S_A(T, \zeta=0) \quad (1)$$

2) Eigenvalue analysis of surface strata

The surface strata are partitioned into n-layers as shown in Figure 1, and a multi-lumped mass model represents its response. The shearing spring K_i and damping coefficient c_i of the i-th layer are given by

$$K_i = G_i / d_i \qquad c_i = h_i G_i T_1 / (\pi d_i) \quad (2\text{-}1)$$
$$m_i = 0.5\,(\rho_i d_i + \rho_{i-1} d_{i-1}) \qquad K_b = 8\, G_b B / (2 - \nu_b) \quad (2\text{-}2)$$

where, G_i, d_i and h_i are shear modulus, thickness and damping factor of the i-th layer and T_1 denotes the first natural period of the surface strata. Elastic spring K_b, which expresses the semi-infinite effect of the engineering bedrock, is attached to the bottom of the model. The first natural period T_1 and the first mode shape U_i (i=1,2,⋯,n) are obtained by solving an eigenvalue equation for the model, then the modal damping factor ζ_1 is evaluated. The mode shape at the ground surface is normalized as $U_1=1$.

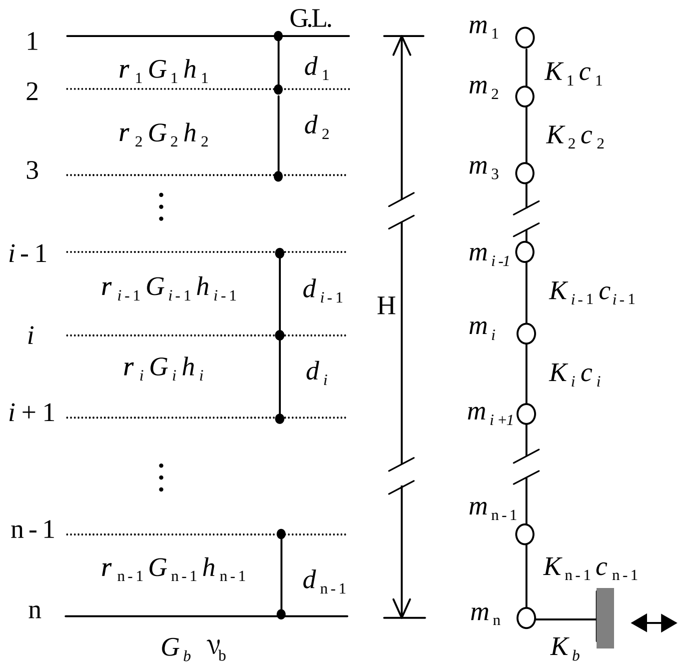

Figure 1: Analytical model

3) Equivalent shear wave velocity Vs_e and impedance ratio α

The surface strata are idealized as a homogeneous stratum with an equivalent shear wave velocity Vs_e, a density ρ_e and a damping factor ζ_1 expressed by Eq. (3).

$$Vs_e = (1/H) \sum_{i=1}^{n-1} Vs_i\, d_i \qquad \rho_e = (1/H) \sum_{i=1}^{n-1} \rho_i\, d_i \quad (3)$$

where, $Vs_i = \sqrt{G_i / \rho_i}$ and $H\ (= \sum_{i=1}^{n-1} d_i)$ is the thickness of the surface stratum.

The impedance ratio α between an engineering bedrock and the surface stratum is expressed by Eq. (4).

$$\alpha = (\rho_e Vs_e) / (\rho_b Vs_b) \quad (4)$$

4) Amplification of surface stratum

The amplification of the surface stratum overlying the engineering bedrock depends on T/T_1, ζ_1 and α, and it is obtained by solving the one-dimensional shear wave propagating equation. The amplification ratios $G_S(T_1, \zeta_1, \alpha)$ at the first natural period T_1, $G_S(T_2, \zeta_1, \alpha)$ at the second T_2 (=T_1/3) on the ground surface, $G_b(T_1, \zeta_1, \alpha)$ at the boundary between the surface stratum and the engineering bedrock (hereafter: the lower boundary) are given by Eqs. (5).

$$G_S(T_1, \zeta_1, \alpha) = \{ \alpha (a_{S1} \zeta_1^2 + b_{S1} \zeta_1 + 1)\}^{-1} \quad (5\text{-}1)$$
$$G_S(T_2, \zeta_1, \alpha) = \{ \alpha (a_{S2} \zeta_1^2 + b_{S2} \zeta_1 + 1)\}^{-1} \quad (5\text{-}2)$$
$$G_b(T_1, \zeta_1, \alpha) = 1 - (a_b \zeta_1^2 + b_b \zeta_1 + 1)^{-1} \quad (5\text{-}3)$$

where,

$$a_{S1} = 1 - (-0.24 \alpha^2 + 1.27 \alpha + 0.03)^{-1} \quad (6\text{-}1)$$
$$b_{S1} = (-0.04 \alpha^2 + 0.61 \alpha)^{-1} \quad (6\text{-}2)$$
$$a_{S2} = (-0.13 \alpha^2 + 0.22 \alpha + 0.03)^{-1} \quad (6\text{-}3)$$
$$b_{S2} = (-0.02 \alpha^2 + 0.21 \alpha)^{-1} \quad (6\text{-}4)$$
$$a_b = 1 - (-0.34 \alpha^2 + 0.79 \alpha + 0.03)^{-1} \quad (6\text{-}5)$$
$$b_b = (0.61 \alpha)^{-1} \quad (6\text{-}6)$$

5) Response acceleration and displacement of surface strata at the first natural period T_1

It is assumed that the input motion is a harmonic wave with a period of T_1 and an amplitude of acceleration Fourier spectrum $F_A(T_1)$. The response accelerations on the ground surface $A_S(T_1)$ and those at the lower boundary $A_b(T_1)$ are given by Eqs. (7-1) and (7-2), and the response displacements on the ground surface $D_S(T_1)$ and those at the lower boundary $D_b(T_1)$ are given by Eqs. (7-3) and (7-4).

$$A_S(T_1) = (1 / T_1)\, G_S(T_1, \zeta_1, \alpha)\, F_A(T_1) \quad (7\text{-}1)$$
$$A_b(T_1) = (1 / T_1)\, G_b(T_1, \zeta_1, \alpha)\, F_A(T_1) \quad (7\text{-}2)$$
$$D_S(T_1) = (T_1 / 2\pi)^2 A_S(T_1) \quad (7\text{-}3)$$
$$D_b(T_1) = (T_1 / 2\pi)^2 A_b(T_1) \quad (7\text{-}4)$$

6) Non-linearity of surface stratum

The relative displacement u_i of the i-th layer from the lower boundary is given by

$$u_i = \{D_S(T_1) - D_b(T_1)\}\, U_i : i=1,2,\cdots,n \quad (8\text{-}1)$$

then, an effective shear strain γ_{ei} of the i-th layer is expressed by

$$\gamma_{ei} = 0.65\, (u_i - u_{i+1}) / d_i \quad (8\text{-}2)$$

where, 0.65 in Eq.(8-2) is a constant of effective strain conversion employed in SHAKE.

The equivalent shear modulus G_{ei} and damping factor h_{ei} corresponding to the effective shear strain γ_{ei} are obtained from the nonlinear relations between shear modulus G, damping factor h and shear strain γ of the i-th layer.

7) Replacement of soil constants

The soil constants of each layer, shear modulus G and damping factor h, are replaced by those corresponding to the shear strains, and the process returns to step 2).

8) Judgement of convergence

The computations from 2) to 7) are iterated until the first natural period T_1 of the subsoil attains a stable value. Finally, the equivalent shear wave velocity Vs_e, the impedance ratio α, the first natural period T_1 of the subsoil and the relative displacement u_i (i=1,2,$\cdots$,n) are obtained.

3. ACCELERATION RESPONSE SPECTRA OF INPUT MOTION

As an example, an acceleration response spectrum $S_{AT}(T, \zeta=0.05)$ shown in Figure 2 is set up as the target spectrum of the input motion. Ten acceleration time histories (hereafter: accelerograms) with different phase characteristics are generated from the target spectrum $S_{AT}(T, \zeta=0.05)$. The acceleration response spectra of a damping factor ζ are calculated for ten accelerograms, and these spectra are averaged at each period T. Then the average acceleration response spectrum $S_A(T, \zeta)$ of the ten accelerograms is formulated by

$$\left.\begin{array}{ll} T \le 0.04 & : S_A(T, \zeta) = 480 \\ 0.04 < T \le 0.14 & : S_A(T, \zeta) = 480+10(a-480)(T-0.04) \\ 0.14 < T \le 0.57 & : S_A(T, \zeta) = a \\ 0.57 < T \le 10 & : S_A(T, \zeta) = a/(cT^{b}) \end{array}\right\} \tag{9}$$

where,

$a = 6278 / (1+19.4\ \zeta^{0.51})$

$b = 1.48 / (1+0.71\ \zeta^{0.13})$

$c = 2.3 / (1+0.42\ \zeta^{0.1})$

In Eqs. (9), $S_A(T, \zeta=0.05)$ corresponds to the target spectrum $S_{AT}(T, \zeta)$ shown in Figure 2. $S_A(T, \zeta=0)$ in Eq.(9) enables to estimate the acceleration Fourier spectrum $F_A(T)$ of the input motion by Eq.(1).

4. APPROXIMATION OF ACCELERATION RESPONSE SPECTRA ON GROUND SURFACE

The acceleration response spectrum $S_{AS}(T, \zeta)$ on the ground surface is simplified by Eqs. (10) using $S_A(T, \zeta)$, $G_S(T_1, \zeta_1, \alpha)$ and $G_S(T_2, \zeta_1, \alpha)$ obtained before. This simplification is introduced from viewpoint of a practical seismic design analysis of building. Figure 3 shows $S_{AS}(T, \zeta)$ at each period.

$$\left.\begin{array}{ll} T_1 \le T & : S_{AS}(T, \zeta) = (T_1/T)^{\beta} S_{AS}(T_1, \zeta) \\ T_2 \le T < T_1 & : S_{AS}(T, \zeta) = S_{AS}(T_2, \zeta) + (T-T_2) / (T_1-T_2) \{ S_{AS}(T_1, \zeta) - S_{AS}(T_2, \zeta)\} \\ T_3 \le T < T_2 & : S_{AS}(T, \zeta) = (T/T_2)\, S_{AS}(T_2, \zeta) \\ T < T_3 & : S_{AS}(T, \zeta) = 500 \end{array}\right\} \tag{10}$$

where,

$S_{AS}(T_1, \zeta) = G_S(T_1, \zeta_1, \alpha)\, S_A(T_1, \zeta)$
$S_{AS}(T_2, \zeta) = G_S(T_2, \zeta_1, \alpha)\, S_A(T_2, \zeta)$
$\beta = \{\log S_{AS}(T_1, \zeta) - \log S_A(T=10, \zeta)\} / (1-\log T_1)$
$T_3 = 500 T_2 / S_{AS}(T_2, \zeta)$

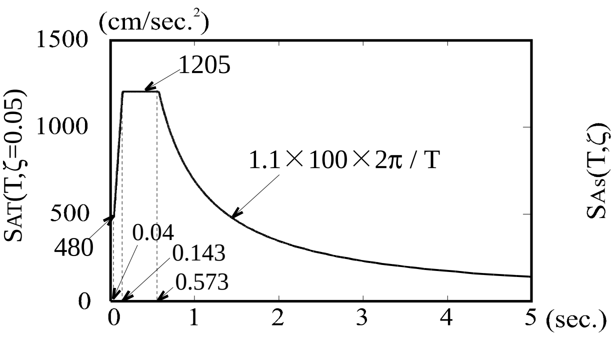

Figure 2: Acceleration response spectra of input motion

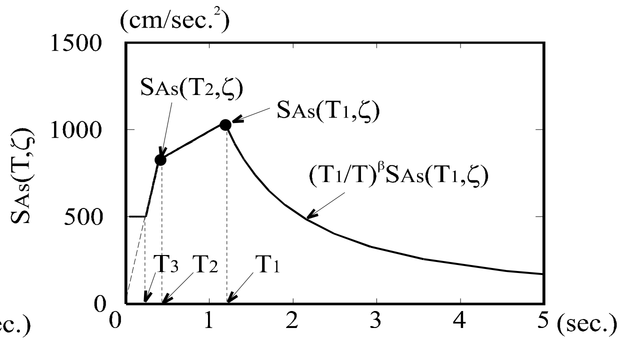

Figure 3: Approximation of acceleration response spectra on the ground surface

5. VERIFICATION OF THE PROPOSED METHOD

To verify the applicability of the proposed method, the nonlinear amplifications of the subsoil are compared with those obtained by SHAKE which employs an equivalent linear analysis. The accuracy of SHAKE is good enough for practical use, and its application provides a more rigorous subsoil response than the proposed method.

Figure 4 shows the initial shear wave velocities of four different kinds of the subsoil, which occur at the objective sites. In this verification, the layer with a shear wave velocity of about 400m/sec. is considered to be the engineering bedrock. The depths of the engineering bedrock at the four sites are G.L.-46.6m for Site-1, G.L.-37.0m for Site-2, G.L.-27.5m for Site-3 and G.L.-9.4m for Site-4. The input motion is set up as an outcrop motion (2Eo) on the engineering bedrock. The acceleration response spectrum $S_{AT}(T, \zeta=0.05)$ shown in Figure 2 is used as the input motion. Ten accelerograms of the input motions, which are necessary for SHAKE, are generated from the target spectrum $S_{AT}(T, \zeta)$ for varying the phases. Nonlinear relations between shear modulus G, damping factor h and shear strain γ are modeled by the Romberg-Osgood model shown in Figure 5. Go in Figure 5 indicates the initial shear modulus.

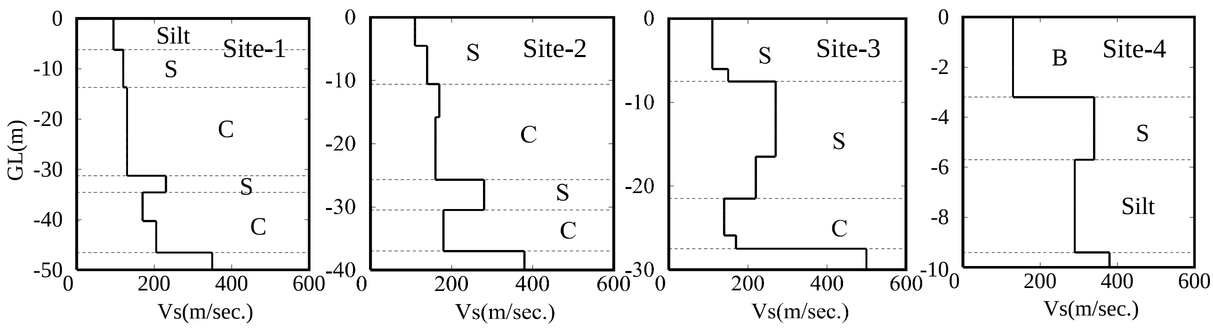

Figure 4: Initial shear wave velocities of the four objective sites (S: Sand, C: Clay, Silt: Silt, B: Burried)

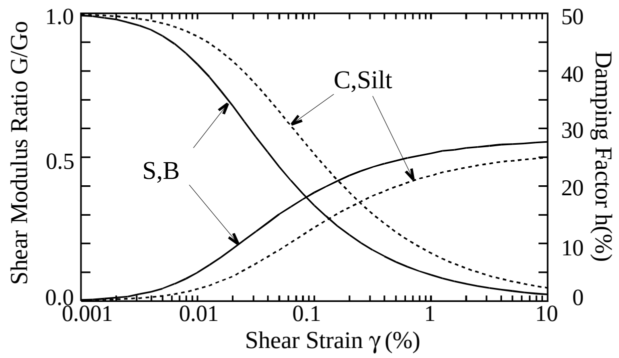

Figure 5: Nonlinear relations between shear modulus Ratio *G/Go*, damping factor *h* and shear strain γ

The amplification ratios between the ground surface and the input motion (2Eo) are compared. Figure 6 compares the amplification ratios obtained by the proposed method with those obtained by SHAKE. The dotted lines show the amplification ratios obtained by SHAKE for ten input motions, and the dashed-and-dotted lines show the amplification ratios in a linear condition with a damping factor of 1%. The natural periods increase and the amplification ratios at the first natural periods decrease due to the non-linearity of the subsoil. Table 1 compares the first natural periods obtained by the proposed method and those obtained by SHAKE. The nonlinear amplifications obtained by the two methods are in good agreement.

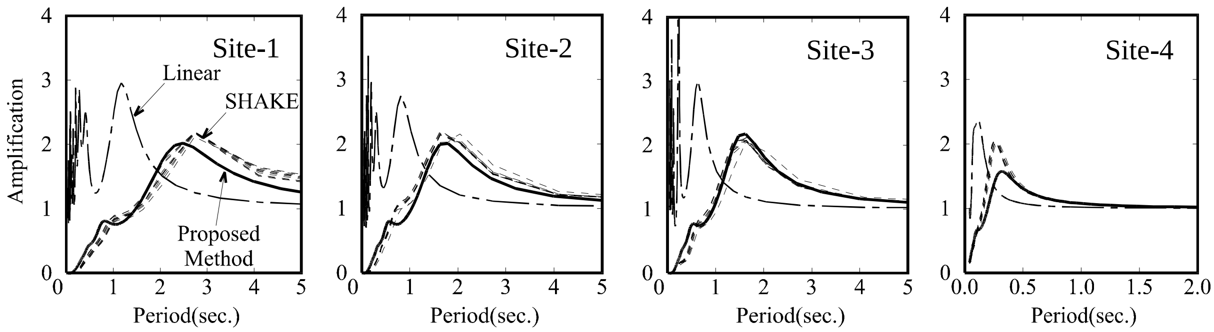

Figure 6: Amplification ratios of ground surface to input motion

Table 1: First natural periods obtained by the proposed method and by SHAKE (unit : sec.)

	Linear condition	Proposed method	SHAKE
Site-1	1.21	2.35	2.55
Site-2	0.84	1.65	1.77
Site-3	0.64	1.54	1.56
Site-4	0.15	0.29	0.27

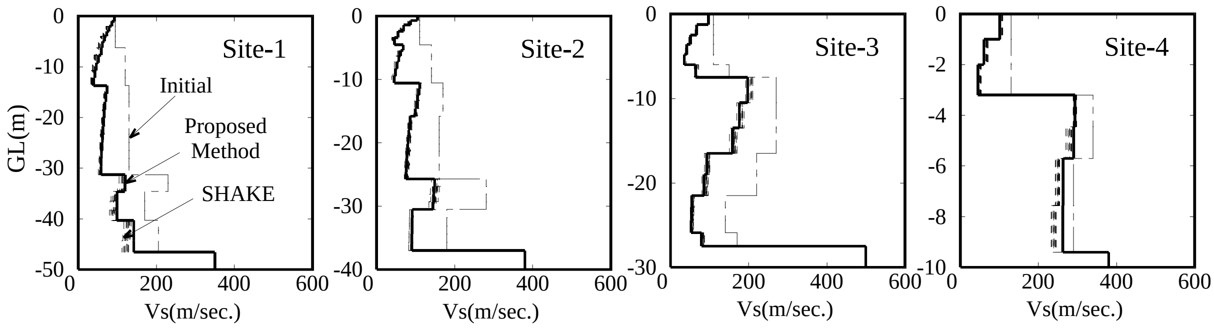

Figure 7: Equivalent shear wave velocities of the subsoil

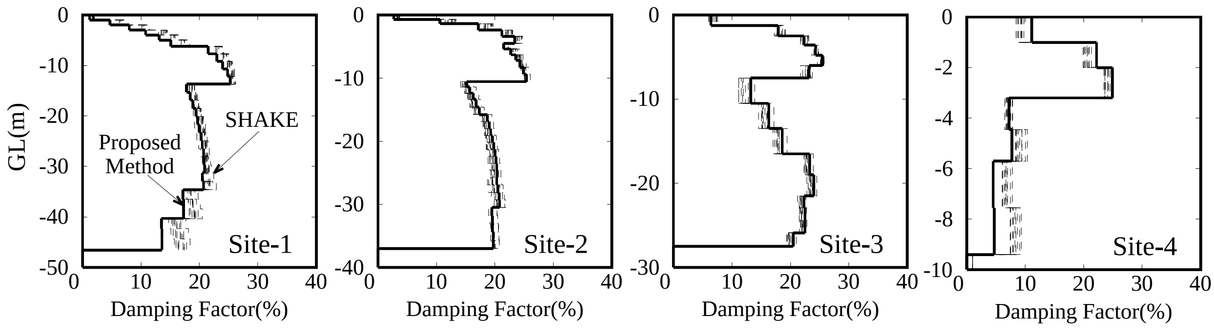

Figure 8: Equivalent damping factors of the subsoil

A dynamic interaction between the building and the subsoil during an earthquake can be represented by dynamic impedance functions. The dynamic impedance functions are calculated using the equivalent shear wave velocities and the equivalent damping factors of the subsoil. Figure 7 and Figure 8 compares the equivalent soil properties, shear wave velocities and damping factors, obtained by the proposed method and by SHAKE. The shear wave velocities decrease and the damping factors increase due to the non-linearity of the subsoil. The equivalent soil properties obtained by the two methods are in good agreement.

The relative displacements of the subsoil are needed to evaluate the pile stresses originating in the dynamic deformation of the subsoil. Figure 9 shows the relative displacements of the subsoil from the engineering bedrock. The relative displacements obtained by the proposed method are calculated by Eq. (8-1), and those obtained by SHAKE are the maximum values. The subsoil displacements obtained by SHAKE exhibit some deviations depending on the differences of the accelerograms. The proposed method yields displacements within the deviation of the results obtained by SHAKE. However, higher than G.L.-3m at Site-4 where the shear wave velocity rapidly changes, the subsoil displacements obtained by the proposed method are greater than those obtained by SHAKE.

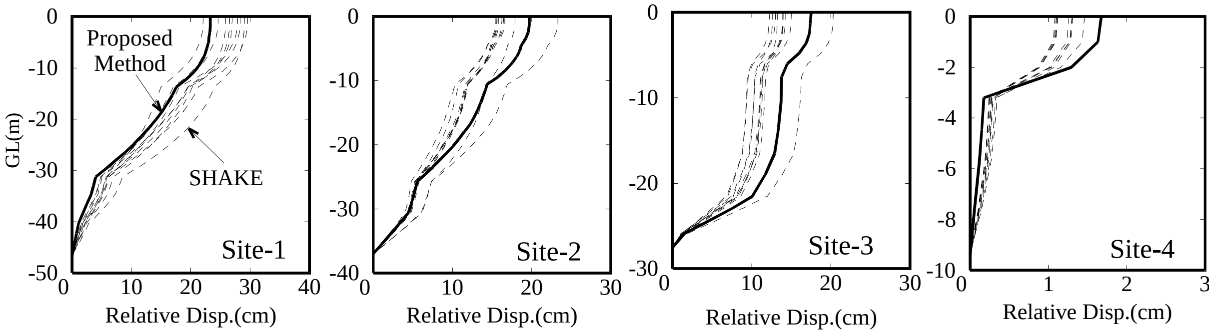

Figure 9: Relative displacements of surface strata

Figure 10 compares the acceleration response spectra on the ground surface with a damping factor ζ of 5%. The acceleration response spectra obtained by SHAKE exhibit some deviations depending on the differences of the accelerograms. The acceleration response spectra obtained by the proposed method almost envelop those obtained by SHAKE. However, the results from Site-4 obtained by the proposed method are less than those obtained by SHAKE in the range of the whole periods. The shear velocity at Site-4 rapidly changes at G.L-3m as shown Figure 8. Care is necessary when applying the proposed method to subsoil like that at Site-4.

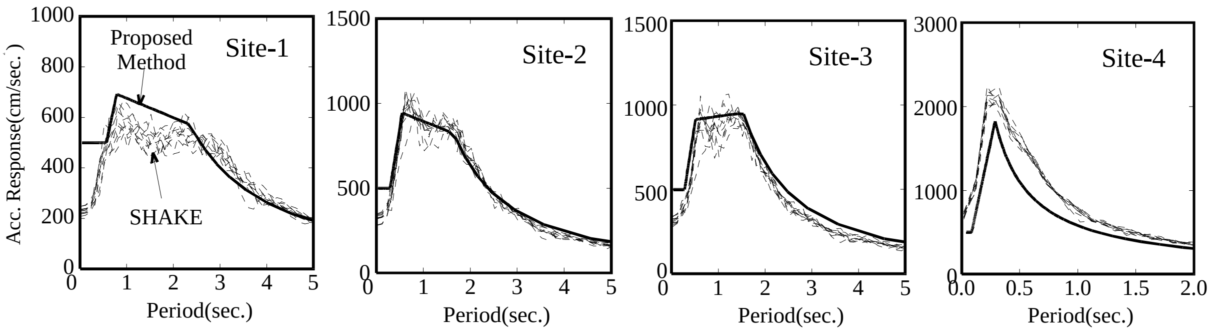

Figure 10: Acceleration response spectra on the ground surface with a damping factor ζ of 5%

6. CONCLUSIONS

A response spectrum method is proposed for evaluating the nonlinear amplification of surface strata overlying the engineering bedrock, and its applicability is examined. Concluding remarks are as follows.

1. The proposed method does not require acceleration waveforms, as do the rigorous methods.
2. The responses and the nonlinear characteristics of the subsoil can be evaluated by the proposed method when the design earthquake input motion is prescribed only in the form of a response spectrum instead of a time history. No significant differences are observed between the results obtained by the proposed method and by the rigorous method.
3. The acceleration response spectrum on the ground surface can be accurately evaluated by the proposed method. However, when the soil constants change rapidly, such as at Site-4, the acceleration response spectra obtained by the proposed method are less than those obtained by SHAKE in the range of the whole periods.

4. The applicability of the proposed method is confirmed from the viewpoint of practical seismic design, if attention is paid to the peculiarity of the proposed method described in the above term 3.

7. ACKNOWLEDGEMENTS

The authors wish to express their appreciation to Dr. T. Kobori, Professor Emeritus of Kyoto University, chief executive advisor of Kajima Corporation, for his insightful suggestions and discussions during the development of this research. Also, the fruitful discussions in a research team, Soil Amplification and Soil-Structure Interaction, Technical Committee on Performance-based Structural Code, Japan Association for Building Promotion, are most appreciated.

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