INFLUENCE OF VIBRATION MODES ON FLUTTER ANALYSIS FOR LONG-SPAN SUSPENSION BRIDGES

Shigeki Kusuhara[1], Ikuo Yamada[2] and Naoki Toyama[3]

ABSTRACT

Since the aerodynamic stability is one of the most important issues in the wind resistance design for the long-span bridges, various studies have been carried out. The studies on the Akashi-Kaikyo Bridge, the world's longest suspension bridge with the center span of 1,991m, revealed that not only the three-dimensional effect of the structure and the wind characteristics but also the influence of multi-vibration modes have to be considered in checking the aerodynamic stability[1)]. If the characteristics of dominant vibration mode changes, the critical flutter velocity also may change. This paper describes the results of flutter analysis for an assumed long-span bridge, which has the center span is approximately 1500m or 2300m, and the influence of vibration mode for each bridge.

1. INTRODUCTION

There are several plans or ideas of strait crossing road projects in Japan (Fig.1)[2)]. In these projects, super long-span bridges, which would be longer than the Akashi-Kaikyo Bridge, are included. In order to make these super long-span bridges to come true, the aerodynamic stability is one of the most important issues.

In the advanced studies on the Akashi-Kaikyo Bridge, special attentions were paid to the following considerations in order to ensure the aerodynamic stability for a long-span bridge.

1) The three-dimensional effect of the structure and the wind
2) The influence of multi-vibration modes

According to the later consideration, it can be possible to improve the aerodynamic stability of long span bridges by controlling the dominant vibration modes. Therefore, the influence of the vibration mode on flutter characteristics of assumed suspension bridges was examined. This study is base on the "Wind resistant design code for Honshu-Shikoku Bridges (2001)[3)]".

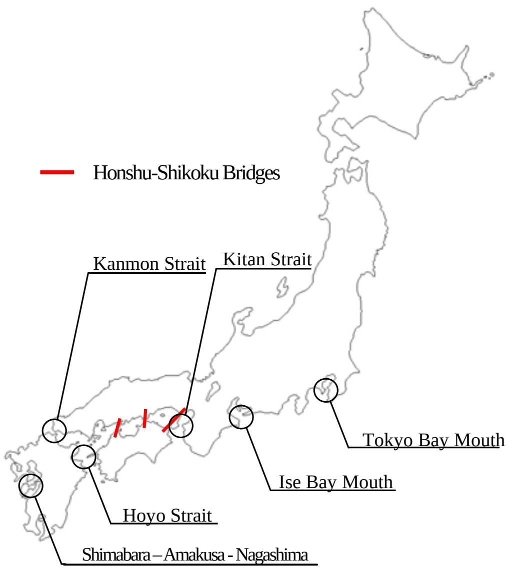

Fig.1 Strait crossing road projects in Japan

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1 Sub-Leader, Wind and Structural Engineering Group, Long-span Bridge Engineering Center, Honshu-Shikoku Bridge Expressway Co., Ltd. (former Honshu-Shikoku Bridge Authority), Japan
2 Leader, ditto
3 Engineer, ditto

2. OUTLINE AND ASSUMPTION

The procedure of the flutter analysis in this paper is shown in Fig.2

Fig.3 shows two suspension bridges assumed in the flutter analysis. These bridges are designed for the above-mentioned strait crossing road projects. For simplicity, a two-span suspension bridge with the center span of 1,480m is called Bridge-A, and a three-span suspension bridge with the center span of 2,250m is called Bridge-B. The cross sections of girders are shown in Fig.4. One-box girder is applied to Bridge-A and slotted-box girder with superior aerodynamic stability is applied to Bridge-B.

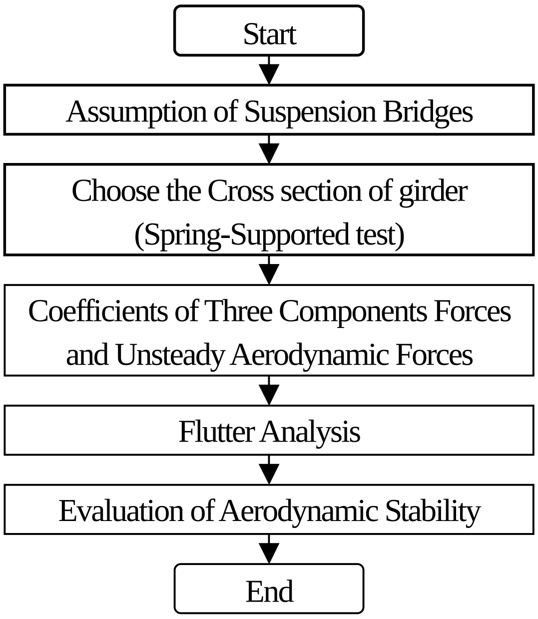
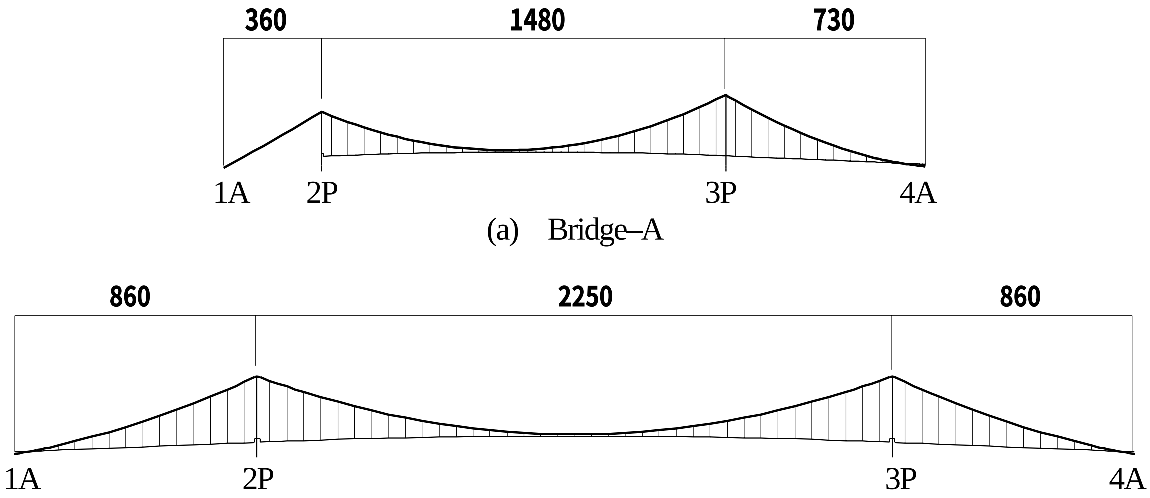
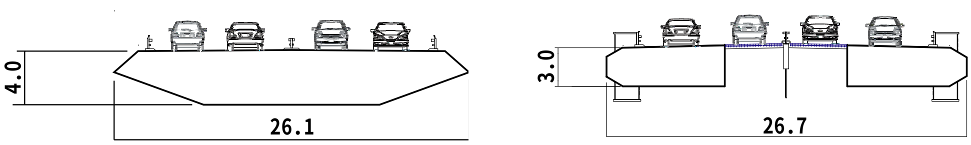


Fig.2 Procedure of flutter analysis



(b) Bridge–B

Fig.3 Analysis objects of assumed suspension bridge (Unit: m)



(a) One-box girder (Bridge-A) (b) Slotted-box girder (Bridge-B)

Fig.4 Cross sections of girders for four lanes (Unit: m)

3. RESULTS OF SPRING-SUPPORTED TEST

The results of the two-dimensional spring supported test at the angles of attack of -3, 0 and +3 degrees are shown in Table 1. Both of the two cross sections showed good aerodynamic stabilities except at +3 degree (one-box girder) and -3 degree (slotted-box girder).

Fig.5 shows coefficients of three components forces. The coefficients of unsteady aerodynamic forces, which coordinate system is defined by Fig.6, is shown in Fig.7. Coefficients of the unsteady aerodynamic forces were defined as follows:

$$L = \pi\rho\left\{B^2\left(L_{ZR}\omega^2 z + L_{ZI}\omega z'\right) + B^3\left(L_{\theta R}\omega^2\theta + L_{\theta I}\omega\theta'\right)\right\} \qquad (1)$$

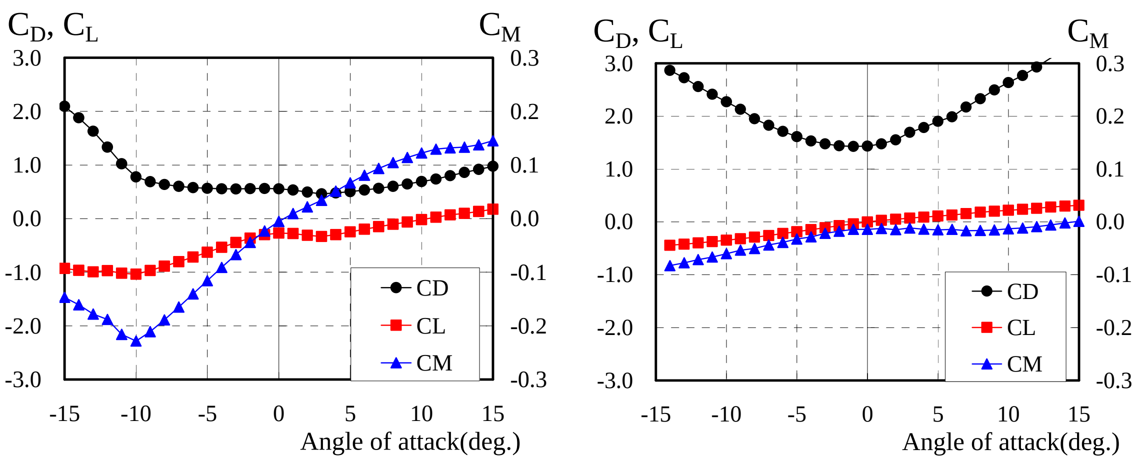
$$M = \pi\rho\left\{B^3\left(M_{ZR}\omega^2 z + M_{ZI}\omega z'\right) + B^4\left(M_{\theta R}\omega^2\theta + M_{\theta I}\omega\theta'\right)\right\} \qquad (2)$$

where, L:lift, M:aerodynamic moment, z:vertical displacement, θ:torsional displacement, ω:circular frequency, ( )':d( )/dt, Lxx or Mxx: coefficients of unsteady aerodynamic forces ($_Z$: caused by vertical vibration, $_\theta$:caused by torsional vibration, $_R$:in phase with displacement, $_I$:in phase with velocity)

Table 1 Results of the two-dimensional spring-supported test

Angle of Attack (deg.)	-3	0	+3
Bridge-A (One-box)	82 m/s+	80 m/s	76 m/s
Bridge-B (Slotted box)	81 m/s	100 m/s+	100 m/s+

Note: "+" indicates that the critical flutter velocity is larger than the tabulated value



(a) Bridge-A (b) Bridge-B

Fig.5 Coefficients of Drag, Lift and Moment

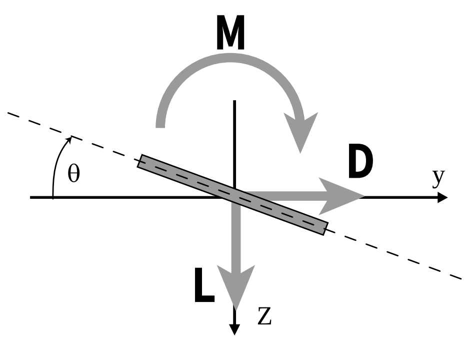


Fig.6 Coordinate system for unsteady aerodynamic forces

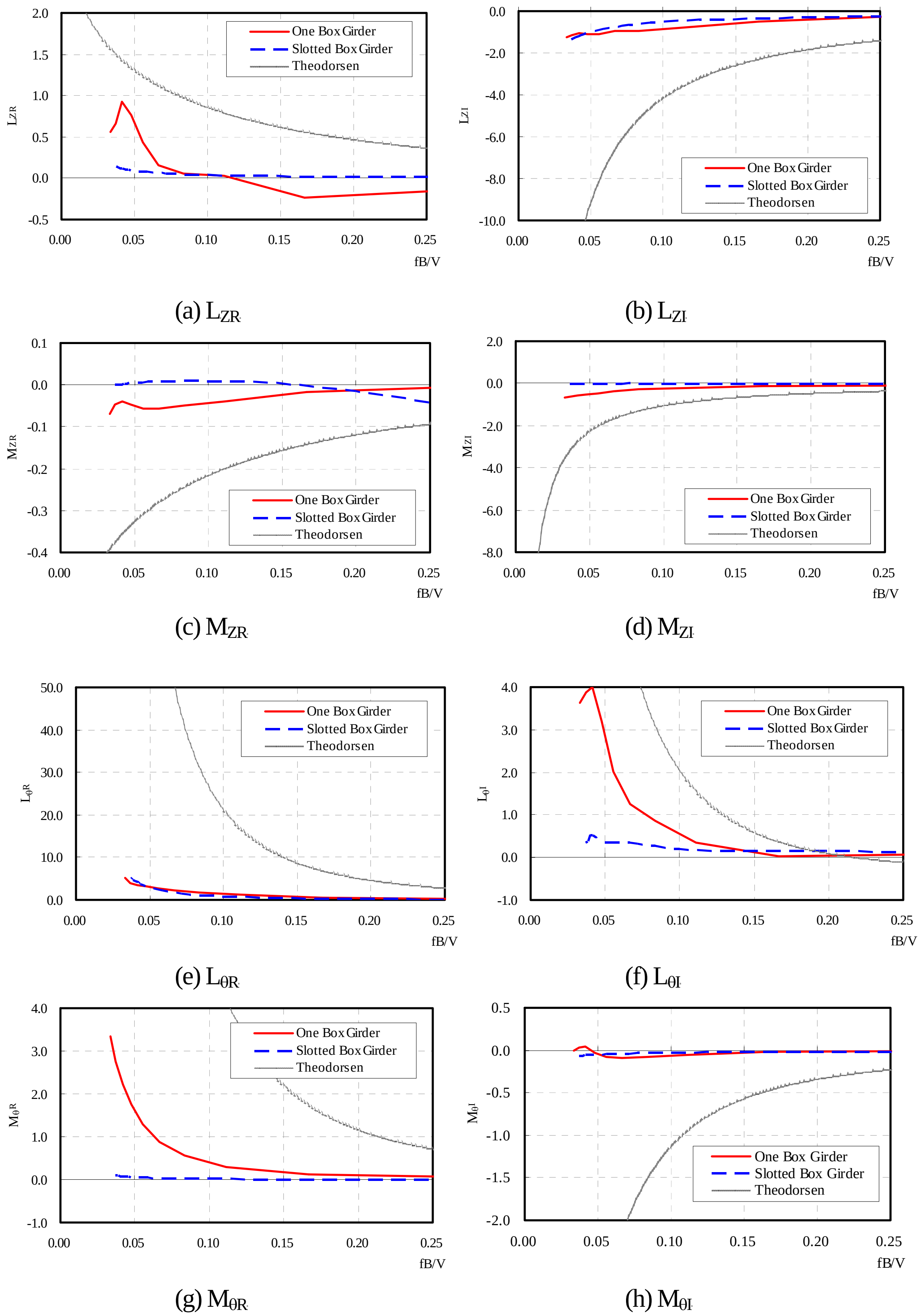


Fig.7 Coefficient of unsteady aerodynamic forces

4. FLUTTER CHARACTERISTICS OF BRIDGE-A

1) Influence of high mode

Before selecting the important modes in flutter analysis, the influence of high natural modes was examined in order to evaluate the number of modes in the analysis. The critical flutter velocities were calculated for Bridge-A with various combinations of natural vibration modes. These calculations applied a multi-mode flutter analysis, using the mode combination method. Static displacements by wind load, which were calculated from the measured three component forces (Fig.5), were also considered.

Table 2 shows the assumption in the flutter analysis. The preliminary analyses by using the lowest 20, 30, 40, and 50 modes were carried out. The analysis concluded that the right solution was equivalent to the approximation by considering at least the lowest 30 modes.

In addition, a mode, which had an influence on the flutter characteristics, was expected to exist between the 20th mode and the 30th mode. Therefore, additional analyses were carried out to identify such mode. The conclusion was that the 21st mode affected largely, and the analysis including modes up to the 22nd mode or more could obtain a good approximation of flutter characteristics of Bridge-A as shown in Fig.8.

Table 2 Assumptions in flutter analysis

<table>
<tr><th>Item</th><th>Analysis condition</th></tr>
<tr><td>Analytical method</td><td>Mode combination method.(Using lower 50 modes.)</td></tr>
<tr><td>Air density</td><td>1.23 kg/m^3</td></tr>
<tr><td>Structural damping</td><td>δ=0.02 for all modes</td></tr>
<tr><td>Static deformation in wind condition</td><td>Considered</td></tr>
<tr><td rowspan="3">Coefficient of aerodynamic forces</td><td>Main girder
<table>
<tr><td></td><td></td><td colspan="3">Direction</td></tr>
<tr><td></td><td></td><td>Vertical</td><td>Torsional</td><td>Horizontal</td></tr>
<tr><td rowspan="3">Forces</td><td>Lift</td><td>○</td><td>○</td><td>△</td></tr>
<tr><td>Moment</td><td>○</td><td>○</td><td>△</td></tr>
<tr><td>Drag</td><td>△</td><td>△</td><td>△</td></tr>
</table>
○;Unsteady aerodynamic forces △;Quasi-steady aerodynamic forces</td></tr>
<tr><td>Cable: Quasi-steady drag force and lift force (C_D=0.7)</td></tr>
<tr><td>Tower: Not considered</td></tr>
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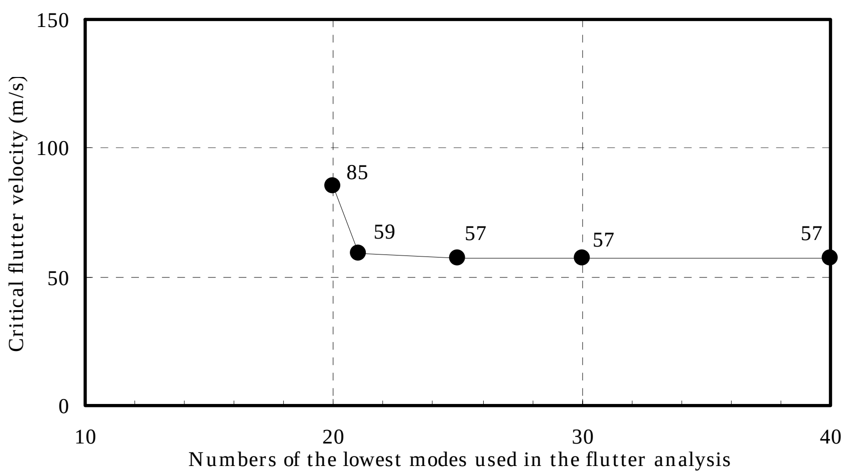


Fig.8 Influence of the lowest natural modes (Bridge-A: angle of attack=+3 deg.)

However, the critical flutter velocities derived from these flutter analyses (Fig.8) were significantly smaller than the results based on the two-dimensional spring-supported test (Table 1). Consequently, it might be difficult for the long-span suspension bridge with center span of 1480m to evaluate the aerodynamic stability by the results of the ordinary two-dimensional wind test.

2) Selection of dominant modes

Since it had been predicted that dominant modes existed between the 1st to the 21st of natural vibration modes, a series of analyses were carried out with various combinations of modes. The analysis resulted that the 2nd (1st symmetric vertical mode), the 8th (2nd symmetric vertical mode), and the 21st (1st symmetric torsional mode) dominate the flutter characteristics of Bridge-A. The combination of these three modes is identified with the combination from the 1st to the 21st mode in the flutter characteristics (shown in Fig.9).

Results of natural vibration analysis without wind load are shown in Table 3. According to the table, both the 20th and the 21st modes are symmetric torsional mode having close frequencies to each other. Based on previous experiences, the 20th mode had been applied to spring-supported test condition because of its lower frequency and smaller equivalent mass. Though, the result indicated that the 21st mode affected more than the 20th mode.

Dominant vibration modes are shown in Fig.10. The 21st mode, which is basically a torsional mode, was combined with not only horizontal mode but also 2nd symmetric vertical mode. Consequently, the 21st mode is considered to have a relatively large contribution to the flutter characteristics. In addition, the proportion of center span length and side span length is exactly 2 to 1 in Bridge-A. This was expected to excite the vertical vibration in high modes.

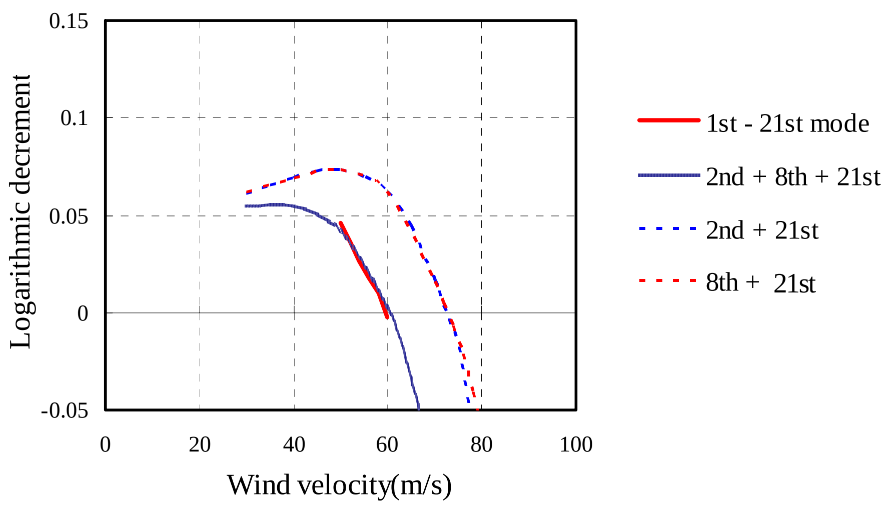
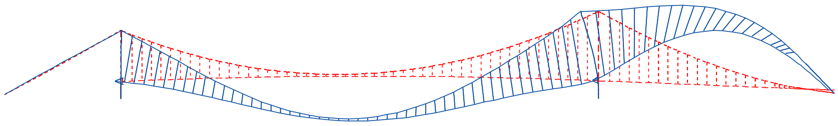
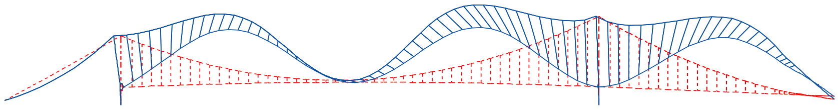


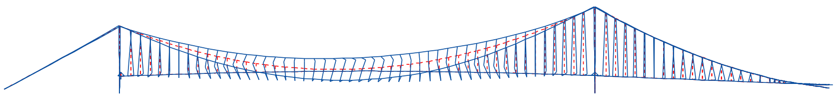
Fig.9 Selection of dominant modes (Bridge-A: angle of attack=+3 deg.)



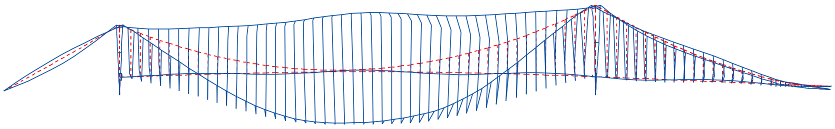
Mode 2 (f=0.078Hz)



Mode 8 (f=0.175Hz)



Mode 20 (f=0.294Hz)



Mode 21 (f=0.295Hz)

Fig.10 Dominant vibration modes (Bridge-A)

Table 3 Natural vibration modes without wind load (Bridge-A)

Mode	Frequency (Hz)	Period (sec)	Equivalent mass (kN/m)				Mode shape
			Longitudinal	Vertical	Horizontal	Torsional	
1	0.046	21.589			166.7		SH-1
2	0.078	12.806		196.1			SV-1
3	0.083	12.025	353.0	304.0			AV-1 (1)
4	0.092	10.893			147.1		AH-1
5	0.115	8.684			156.9		AH-2
6	0.116	8.592	264.8	382.5			AV-1 (2)
7	0.132	7.569	4020.7	186.3			SV-2 (1)
8	0.175	5.711		176.5			SV-2 (2)
9	0.182	5.485			156.9		SH-2 (1)
10	0.186	5.370		176.5			SV-2 (1)
11	0.193	5.177		186.3			AV-2 (2)
12	0.220	4.552			2177.1		
13	0.228	4.395					
14	0.243	4.113		176.5			SV-3
15	0.245	4.081			1147.3		SH-2 (1)
16	0.252	3.974					
17	0.263	3.800					
18	0.265	3.769			892.4		SH-2 (2)
19	0.272	3.674			1304.3		
20	0.294	3.401			490.3	18485.5	ST-1 (1)
21	0.295	3.391			294.2	38079.2	ST-1 (2)
22	0.295	3.389		176.5			AV-3 (1)
23	0.305	3.277		205.9			AV-3 (2)
24	0.324	3.082		1088.5			
25	0.334	2.993			509.9		

S: symmetric, A: asymmetric, H: horizontal, V: vertical ,T: torsional

5. FLUTTER CHARACTERISTICS OF BRIDGE-B

1) Influence of higher mode

The influence of high natural modes was examined in order to evaluate how many number of modes in the analysis for Bridge-B in the same way as Bridge-A. The analysis including modes up to the 20th mode could obtain a good approximation of flutter characteristics of Bridge-B as shown in Fig.11. Therefore, the influence of the lowest 20 modes of Bridge-B was investigated.

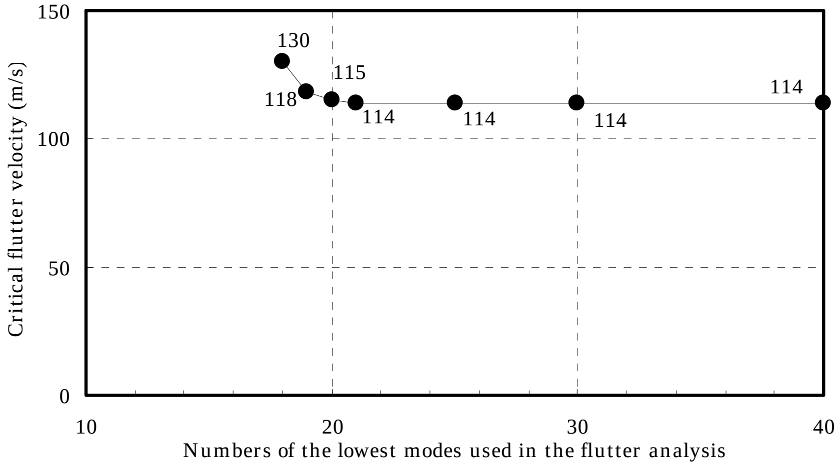


Fig.11 Influence of the lowest natural modes (Bridge-B: angle of attack=0 deg.)

2) Selection of dominant modes

Flutter analyses with various combinations of vibration modes of Bridge-B were carried out by the same way as Bridge-A. Fig.12 shows the result of analysis. The combination of the 4th mode (1st symmetric vertical mode) and the 13th mode (1st symmetric torsional mode), which usually applied to spring supported test, did not excite the flutter. Furthermore, the result of flutter analysis with the lowest 20 modes can be represented by the analysis with four dominant modes, 4th, 11th (2nd symmetric vertical mode-a), 13th and 14th (2nd symmetric vertical mode-b). The critical flutter velocity, calculated by removing the 11th mode from these four modes, was lower than that calculated with these four modes.

As mentioned above, it was found that the 11th mode is the important mode which suppresses the excitation of flutter characteristics.

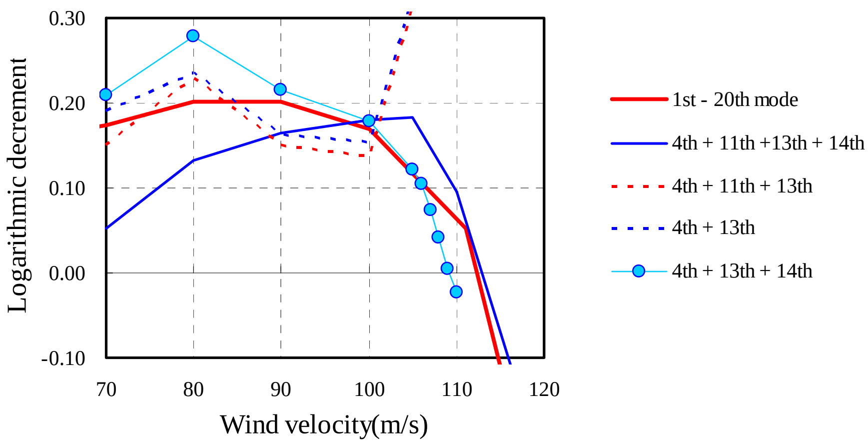


Fig.12 Selection of dominant modes (Bridge-B: angle of attack=0 deg.)

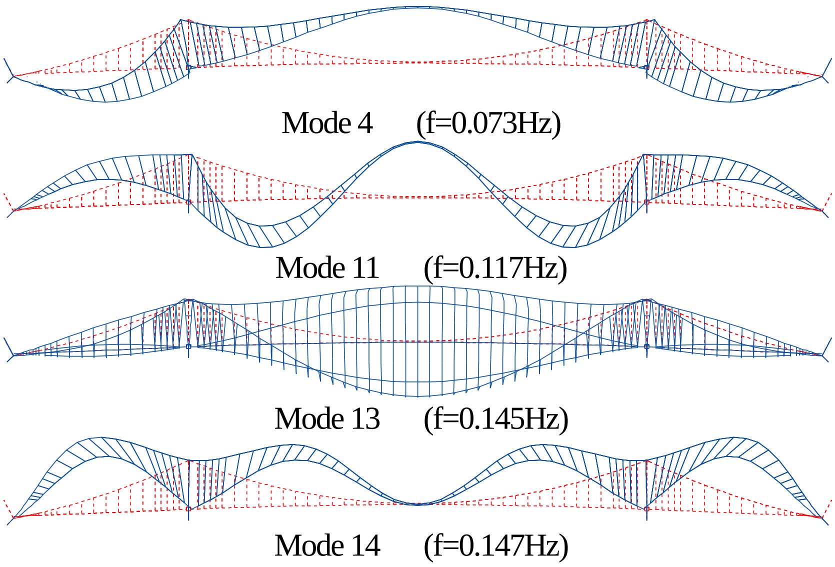


Fig.13 Dominant vibration modes (Bridge B)

Table 4 Natural vibration modes without wind load (Bridge-B)

Mode	Frequency (Hz)	Period (sec)	Equivalent mass (kN/m)				Mode shape
			Longitudinal	Vertical	Horizontal	Torsional	
1	0.037	26.851			19.6		SH-1
2	0.066	15.084			19.6		AH-1
3	0.072	13.924	49.0	29.4			AV-1 (1)
4	0.073	13.661	509.9	19.6			SV-1
5	0.084	11.875	19.6	323.6			L
6	0.086	11.692	19.6	470.7			L
7	0.086	11.611			19.6		SH-1 (1s)
8	0.086	11.610			19.6		SH-1 (2s)
9	0.098	10.168	49.0	39.2			AV-1 (1)
10	0.103	9.717			19.6		SH-2 (1)
11	0.117	8.544	4520.9	19.6			SV-2 (1)
12	0.118	8.502	39.2	39.2			AV-1 (2)
13	0.145	6.904			3834.4	2226.1	ST-1
14	0.147	6.794	5403.5	19.6			SV-2 (2)
15	0.156	6.411			3		AH-2 (1)
16	0.157	6.353	50720.0	19.6			AV-2
17	0.165	6.078					
18	0.171	5.850					
19	0.176	5.687			107.9	229642.3	SH-2 (2)
20	0.178	5.615			78.5	4118.8	AT-1 (1)
21	0.181	5.530			68.6	5364.2	AT-1 (2)
22	0.194	5.162			205.9		SH-2 (3)
23	0.194	5.152			137.3		AH-2 (2)
24	0.198	5.059	43257.1	19.6			SV-3
25	0.198	5.056					

S: symmetric, A: asymmetric, H:horizontal, V: vertical T: torsional, L: longitudinal, (s):side-span mode

6. EVALUATION ON ENERGY

Besides three-dimentional analysis, the investigation was carried out in order to identify the part of span exciting the flutter. The energy of aerodynamic forces which work on the girder was examined.

The energy excited by aerodynamic forces are defined in the following equation.

$$W_L = \oint L_R dy_R \qquad W_M = \oint M_R d\phi_R \tag{3}$$

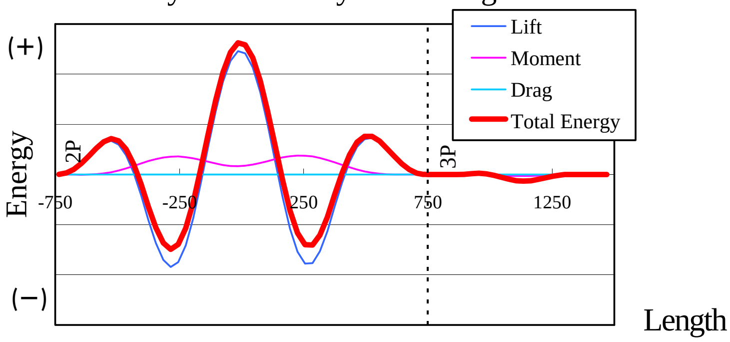
where W_L and W_M are the energy in vertical and torsional directions, $\oint$ is the path integral during vibration, L_R and M_R are the real part of the lift and pitching moment of unsteady aerodynamic forces, y_R and and ϕ_R are the real part of the vertical and torsional displacements, respectively.

The aerodynamic stability of the bridge depends on plus or minus of Eq.(4), the integral of energy on each nodal points.

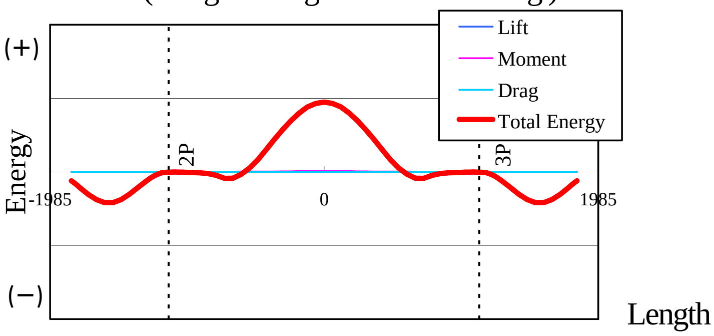
$$\int_0^{span} W dl = \int_0^{span} (W_L + W_M) dl \tag{4}$$

Fig.14 shows the spanwise distributions of energy on girders of Bridge-A and Bridge-B at the critical flutter velocity. It is assumed that the spanwise distribution of energy is strongly associated with the flutter mode shape (shown in Fig.15). When we focus the energy distribution in the center span, Bridge-A indicates a high symmetric mode (the 3rd symmetric mode). On the other hand, Bridge-B is the 1st symmetric mode shape. Therefore, each bridge is evaluated to have a different characteristic.

Furthermore, the distribution of energy in the side span, as shown in Fig.14, seems to be minus, and it means that the side span has a damping effect regularly. Bridge-A has only single side span, and this fact seems to reduce the aerodynamic stability of the Bridge-A.

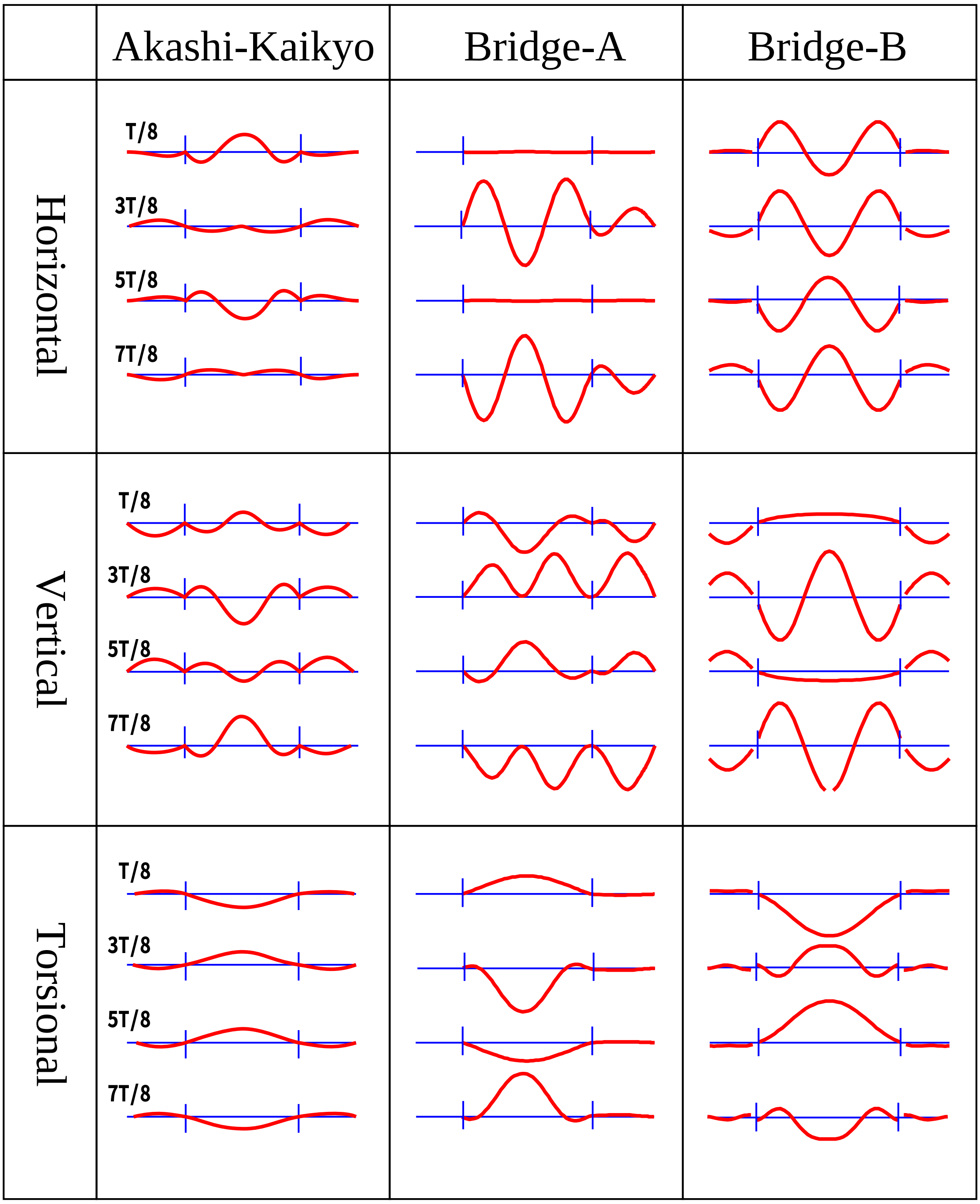


(Bridge-A: angle of attack=+3 deg.)



(Bridge-B: angle of attack=0 deg.)

Fig.14 Spanwise distribution of energy



T: period

Fig.15 Flutter mode shape

7. CONCLUSIONS

The flutter analyses were carried out for the assumed long-span suspension bridges with the center spans of 1,500m and 2,300m. The results of the flutter analyses are summarized as follows,

1) The combination of the lowest flexural mode and torsional mode, which usually used in the two-dimensional spring supported test, might have the possibility of underestimating the flutter instability for super long-span bridges.
2) The characteristics of multi-mode flutter with dozens of vibration modes can be represented by the analysis with only three or four dominant modes in flutter analysis.
3) Some of the dominant modes can control the excitation of flutter, which implies that the adjustment of vibration modes can enhance the critical flutter velocity.

It is necessary to obtain the influence of high natural modes in more detail by using the same analytical method for other suspension bridges.

REFERENCES

1) Honshu-Shikoku Bridge Authority: “*The Akashi-Kaikyo Bridge: Design and Construction of the World’s Longest Bridge”*, pp.97-100, 1999.8
2) H. Sato, N. Hirahara, K. Fumoto, S. Hirano and S. Kusuhara: *“Full Aeroelastic Model Test of a Super Long-span Bridge with Slotted Box Girder”*, Journal of Wind Engineering and Industrial Aerodynamics, No.90, pp.2023-2032, 2002
3) Honshu-Shikoku Bridge Authority: “W*ind resistant design code for Honshu-Shikoku Bridges (2001)”*, 2001.8
4) N. Toyama, K. Hata and S. Kusuhara: *“The Influence of the Vibration Mode on Flutter Characteristics of Long-span Bridges”*, The 6th Asia-Pacific Conference on Wind Engineering, 2005,9