

# Real-time Hybrid Vibration Experiments with a 2-Degrees-of-Freedom Model

by

Keiichi TAMURA<sup>1)</sup> and Hiroshi KOBAYASHI<sup>2)</sup>

## ABSTRACT

The real-time hybrid vibration experiment is a new concept experiment combining shaking table test and numerical response analysis in real-time. Real-time hybrid vibration experiments were conducted with a 2 degrees-of-freedom model. Results of experiments were compared with those of conventional shaking table tests, and the validity of real-time hybrid vibration experiment is demonstrated.

**Key words:** real-time hybrid vibration experiment, shaking table test, pseudo dynamic experiment, actuator delay compensation

## 1. INTRODUCTION

Most of the conventional hybrid experiments are pseudo dynamic experiments with an expanded time axis due to limitations of devices, e.g., capability of computation and compensation for actuator delay, and few precedents exist for hybrid vibration experiments using shaking tables. Differing from those conventional techniques, the real-time hybrid vibration experiment is a new concept experiment combining shaking table test and numerical response analysis in real-time.

We have developed a real-time hybrid experiment system at the Public Works Research Institute after the 1995 Kobe Earthquake. Presented are

the results of real-time hybrid vibration experiments with a 2 degrees-of-freedom model. Results are compared with those of shaking table tests and the validity of real-time hybrid vibration experiment is demonstrated.

## 2. OVERVIEW OF REAL-TIME HYBRID VIBRATION EXPERIMENT

As Figure-1 shows, in the real-time hybrid vibration experiment, an original structure is divided into two parts. One is an actual model specimen of original structure. This specimen is usually taken as a part of structure whose seismic behavior is unknown or complicated. The other is a numerical model for vibration response analysis. This model represents a part of structure whose seismic behavior can be evaluated by numerical analysis.

The numerical model consists of structural elements (mass, damping and stiffness matrices), external force, which is calculated from the acceleration of shaking table, and reaction force

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1)Head, Ground Vibration Division, Earthquake Disaster Prevention Research Center, Public Works Research Institute, Ministry of Construction, Asahi 1, Tsukuba-shi, Ibaraki-ken, 305-0804, Japan.

2)Research Engineer, ditto.

generated at the boundary of the actual and numerical models. In the numerical analysis, the external and reaction forces are inputted, and the displacement of the actual model for the next time step is calculated. This displacement is realized by actuators. Then, the external and reaction forces are measured and taken into numerical analysis. Iterating these procedures, the seismic behavior of original structure can be accurately simulated.

### 3. FEATURES OF REAL-TIME HYBRID VIBRATION EXPERIMENT

#### Time Interval of Experiment

It takes short time from measuring reaction force to generating actuator signal for the next time step<sup>1)</sup>. A central difference method is employed for numerical analysis. Time required for one cycle process is 2.08ms.

#### Actuator Delay Compensation

Since the response delay is inevitable with a hydraulic actuator, a compensation technique is adopted. This technique predicts the displacement of an actuator at the time after actuator delay  $\delta t$ <sup>2)</sup>. A schematic illustration of compensation process is presented in Figure-2, in which the predicted displacement  $x''$  is estimated from the calculated displacement  $x_j$  using an  $n$ th-order polynomial equation (1):

$$x'' = \sum_{j=0}^3 a_j x_j \quad (1)$$

where

$$a_0=4, a_1=-6, a_2=4, a_3=-1$$

The predicted displacement is modified so that the resulting displacement becomes equal to the calculated one.

### 4. HYBRID VIBRATION EXPERIMENTS WITH A 2-DEGREES-OF-FREEDOM MODEL

In order to demonstrate the validity of real-time hybrid vibration experiment, both real-time hybrid vibration experiments and conventional experiments with a whole structure model were conducted, and the results were compared.

The original structure for experiment is a 2 storied slab model supported by rubber bearings. Its conceptual view is presented in Figure-1. The slab part consists of H-shaped beams and infilled concrete. An advantage of rubber bearing is large bearing capacity and low rigidity. Using rubber bearings and heavy slabs makes it possible to prolong the natural period of test specimen, which is favorable for experiment. This test specimen basically represents a linear structure, however, nonlinear behavior of structure is also represented by installing a viscous damper between upper and lower slabs or between the shaking table and the lower slab.

Test cases of hybrid vibration experiments are listed in Table-1. In the hybrid vibration experiments, the mass, stiffness and damping of the upper level and the mass of the lower level were numerically modeled, as shown in Figure-1. Pseudo dynamic experiments with 50 times extended time axis were carried out to demonstrate the effects of experimental time axis on damping force estimation, which generally depends on velocity. Besides those hybrid vibration experiments, conventional experiments with the whole structure were also conducted, and the results of hybrid vibration experiments and conventional experiments were compared.

Input motions employed for experiments were five cycles of sinusoidal wave with the frequency

of 1.7Hz, which corresponds to the fundamental natural frequency of the whole structure, and the strong ground motion recorded at Kobe Maritime Observatory, Japan Meteorological Agency, during the 1995 Kobe Earthquake. The maximum peak acceleration was variously changed in experiments.

## 5. NUMERICAL MODELS FOR HYBRID VIBRATION EXPERIMENTS

### 5.1 Linear structure (Cases 1 and 2)

The equation of motion for numerical analysis is described by

$$M\ddot{x} + C\dot{x} + Kx = p + q \quad (2)$$

where

$M$ : Mass matrix

$C$ : Damping matrix

$K$ : Stiffness matrix

$x$ : Relative displacement vector

$p$ : external force (seismic response) vector

$q$ : reaction force vector

The mass, damping and stiffness were determined as follows:

#### Mass

Mass of slabs and rubber bearings were considered.

#### Damping

The damping matrix was estimated from the damping ratio to the critical  $h=0.026$ , which was obtained by free vibration tests, and the relationship  $C = \beta K$  ( $\beta = 2h/\omega$ ).

#### Stiffness

The relationship between load and displacement was obtained by forced vibration tests, where the frequency was set as 0.1Hz and

1.7Hz. The result is shown in Figure-3. Since the maximum peak displacement of the upper slab in experiments with the whole structure was 20-30mm, the stiffness coefficient was determined as  $2.6 \times 10^6 \text{ N/m}$ .

### 5.2 Upper nonlinear structure (Cases 3 and 4)

#### Mass

Same as 5.1.

#### Damping and Stiffness

The damping and stiffness matrices can not be developed by usual matrix operation, when nonlinearity exists. In the present study, the reaction force vector  $q$  was divided into a linear part  $q_1 (=Kx)$  and nonlinear part  $q_2$  as shown in Figure-4.  $q_1$  and  $q_2$  can be estimated from measuring reaction force and nonlinear calculation, respectively. Since the damping force generally depends on velocity, the relationship between force and velocity was estimated from the forced vibration tests of dampers, which is shown in Figure-5. Figure-6 presents the relationship between the damping force and velocity. As seen from this figure, this damper generates force almost proportional to the velocity for the pull-side, and small and velocity independent force for the push-side.

### 5.3 Lower nonlinear structure (Cases 5 to 8)

Same as 5.1.

## 6. EXPERIMENT RESULTS

Figure-7 compares displacement time histories of the lower slab for hybrid vibration experiments and conventional shaking table tests with the whole structure, where the results of pseudo dynamic hybrid experiments are plotted with the actual time axis. The cross-correlation coefficients between hybrid vibration

experiments and conventional shaking table tests are calculated for displacements and shown in Table-2. The following points may be deduced from the experiment results.

- (1) The real-time hybrid vibration experiment well reproduces the seismic behavior of the 2-degrees-of-freedom structure for both linear and nonlinear cases.
- (2) The pseudo dynamic hybrid experiment yields excessive response and inappropriate response period. This may be attributed to the fact that pseudo dynamic experiment can not properly reproduce the damping force, which is generally proportional to velocity.
- (3) The cross-correlation coefficients between the calculated and resulting displacements in hybrid vibration experiments show very high values, such as higher than 0.99, for all cases, which indicates that actuator delay compensation successfully works.
- (4) Causes of slight error between real-time hybrid vibration experiments and conventional shaking table tests may exist in:
  - a. Effects of noise associated with measuring instruments.
  - b. Effects of high-frequency forced vibration of shaking table caused by actuators for hybrid vibration experiment.
  - c. Slight difference of input motions for real-time hybrid vibration experiments and conventional shaking table tests.
  - d. Error in establishing parameters for numerical analysis model.

## 7. CONCLUSIONS

The main conclusions of the present study may be summarized as follows:

- 1) The technique employed in this study to compensate actuator delay has sufficient accuracy.
- 2) The real-time hybrid vibration experiment well reproduces seismic behavior of the 2 degrees-of-freedom model and the validity of experiment is confirmed.

## REFERENCES

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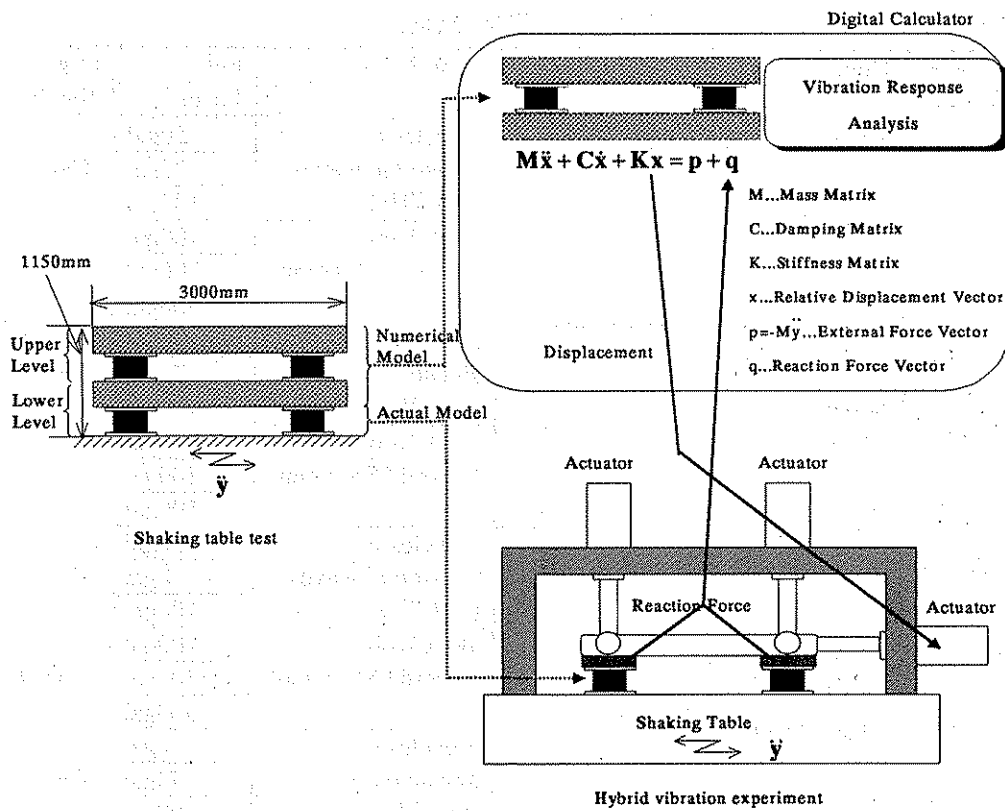


Figure-1 Conceptual view of hybrid vibration experiment

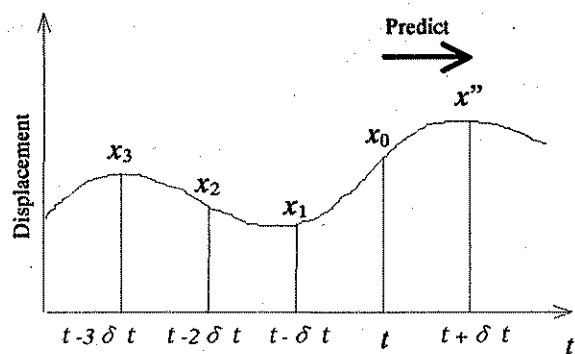


Figure-2 Calculation of predicted displacement  $x''$

Table-1 Test cases

Experiment Method	Upper Level	Lower Level	Input Motion	Input Level	Case Number
Real-time	Linear Model	Linear Model	5 cycles of sinusoidal waves ( $f=1.7\text{Hz}$ )	50gal	Case 1
				70gal	
				100gal	
			JMA Kobe Record (NS comp)	100gal	Case 2
				300gal	
				500gal	
	Nonlinear Model	Linear Model	5 cycles of sinusoidal waves ( $f=1.7\text{Hz}$ )	50gal	Case 3
				70gal	
				100gal	
			JMA Kobe Record (NS comp)	100gal	Case 4
				300gal	
				500gal	
Pseudo Dynamic	Linear Model	Nonlinear Model	5 cycles of sinusoidal waves ( $f=1.7\text{Hz}$ )	50gal	Case 5
				70gal	
				100gal	
			JMA Kobe Record (NS comp)	100gal	Case 6
				300gal	
				500gal	
			5 cycles of sinusoidal waves ( $f=1.7\text{Hz}$ )	50gal	Case 7
				70gal	
			JMA Kobe Record (NS comp)	100gal	Case 8
				300gal	
				500gal	

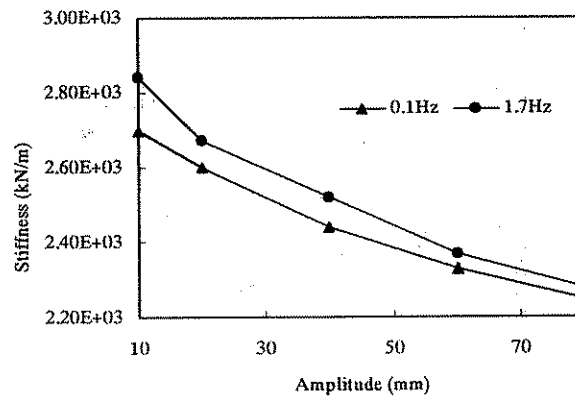


Figure-3 Relationship between amplitude and stiffness

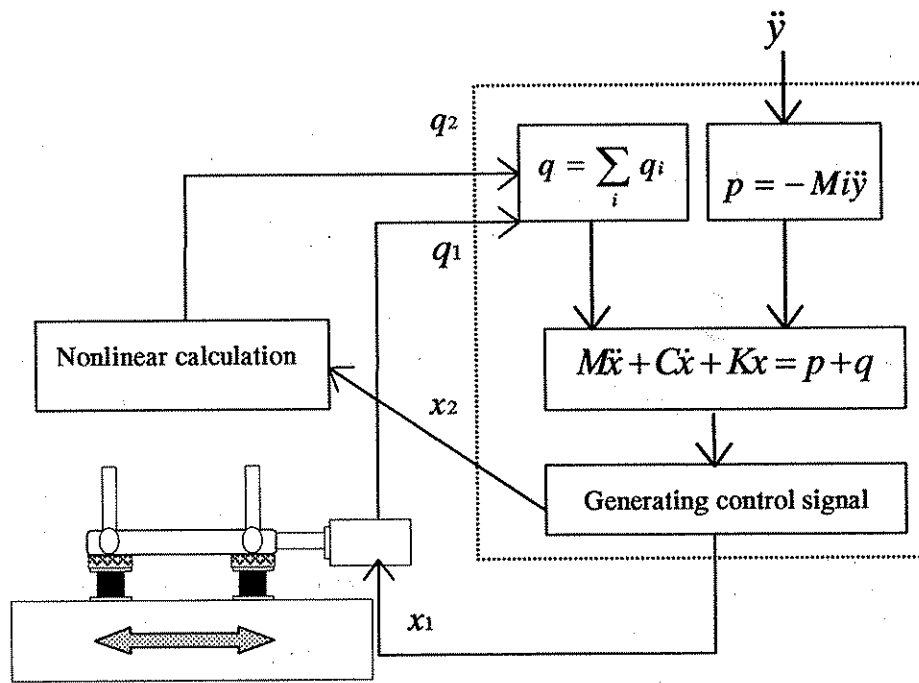


Figure-4 Hybrid experiment dealing with nonlinear factor in vibration response analysis

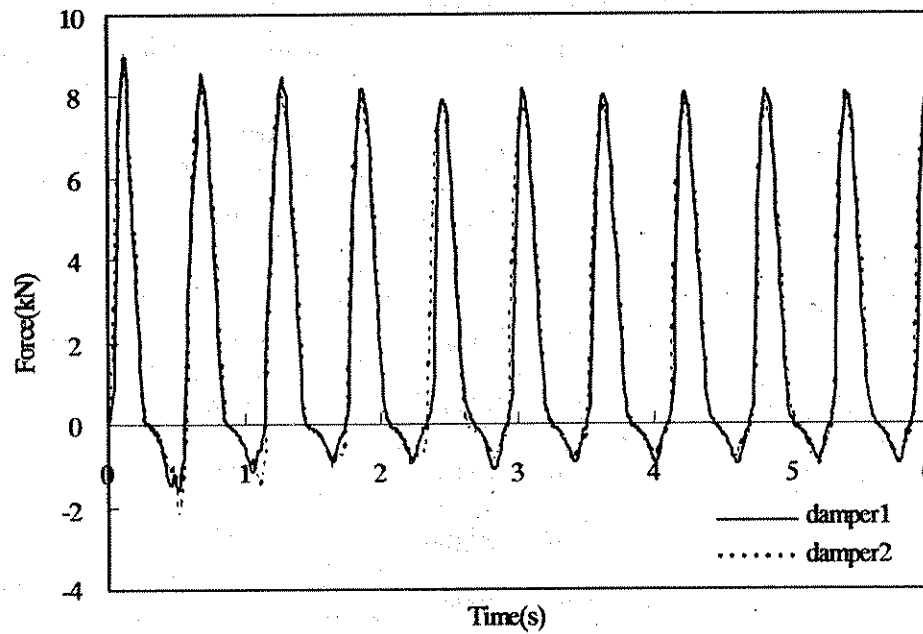


Figure-5 Forced vibration test of damper  
(Sinusoidal wave,  $f=1.7\text{Hz}$ , Amplitude=80mm)

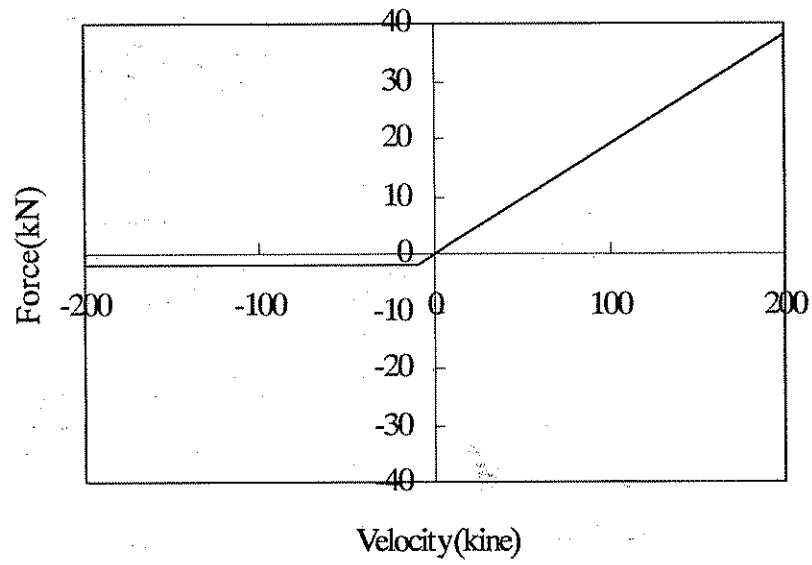


Figure-6 Relationship between damping force and velocity

Table-2 Correlation coefficients between hybrid vibration experiments and shaking table tests

Case	Correlation coefficient
Case1 100gal	0.966
Case2 500gal	0.896
Case3 100gal	0.881
Case4 500gal	0.980
Case5 100gal	0.981
Case6 500gal	0.938
Case7 100gal	0.776
Case8 500gal	0.829



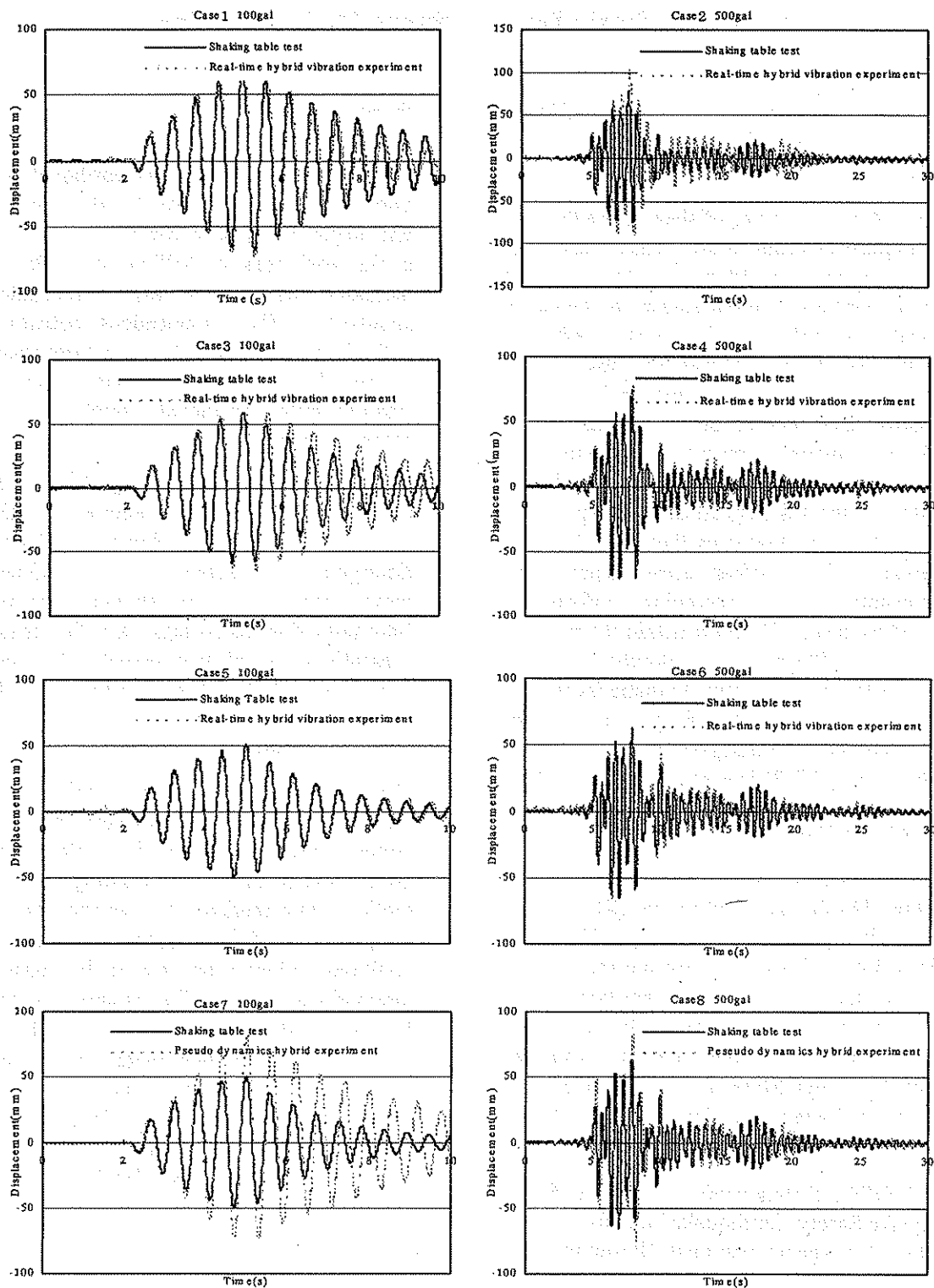


Figure-7 Displacement time histories of the lower slab