Equivalent-Linear Ground-Response Calculations with Frequency-Dependent Damping

by

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ABSTRACT

Equivalent-linear procedures are widely used for calculating the response of soils to earthquake ground motion. They are not the only methods for this purpose; fully nonlinear calculation methods are available, as well as methods relying on empirical analysis of existing strong-motion data. At high levels of input motion, equivalent-linear ground-response calculations for sites in the Imperial Valley of California predict much greater depletion of high-frequencies in the Fourier amplitude spectrum than found for observed data or for calculations performed using fully nonlinear computer codes. Such behavior should be expected if, as suggested many years ago by Dobry, the motion produced by a small cyclic load superimposed on a large load suffers less damping than the motion produced by the large load. Strictly speaking, of course, the effects of two different loads can not be superimposed if the response is nonlinear. The approximate validity of superposition must be assumed, however, if equivalent-linear procedures are to be used at all. Since the spectrum of strain amplitude (which is similar in shape to the spectrum of particle velocity) decreases above the corner frequency of the earthquake approximately as the reciprocal of frequency, the strain-dependent damping used for equivalent-linear calculations should decrease with frequency above the corner frequency, rather than be constant over frequency as in the conventional equivalent-linear procedure. As recently proposed by Sugito, equivalent-linear procedures may be improved by allowing frequency-dependent properties at high strain. I show comparisons of response spectra of motions resulting from fully nonlinear calculations and equivalent-linear calculations using the conventional procedure and a modified procedure in which the damping at high frequency decreases as the reciprocal of frequency. The results of the modified equivalent-linear procedure agrees better with the results of the fully nonlinear calculations than the results of the conventional equivalent-linear procedure, for a site with zero low-strain damping. For a modest amount of low-strain damping, however, the differences between the response spectra for the conventional and modified equivalent-linear results is small, suggesting that conventional equivalent-linear procedures are adequate for many engineering purposes.

KEYWORDS: engineering seismology; equivalent-linear; nonlinear; soil response; strong motion

1. INTRODUCTION

Equivalent-linear methods are widely used in soil-response calculations. In the conventional application, response is calculated as if the soil material were linear, but the shear modulus and damping are chosen to correspond with the strain in each soil layer, determined by iterative calculation. Curves of shear modulus and damping as a function of strain are obtained from laboratory measurements.

For sites where strong-motion records were obtained in the 1979 Imperial Valley, California, earthquake, Durward et al. (1996) found that

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small. The reinforcements of the pile heads yielded. The damage is caused not only by the inertial force of the superstructure and but also by the deformation of surface layer. Third state when 14-20 seconds, the upper part of the soil is liquefied, although the lower part do not completely liquefy. The process of liquefaction cause large relative displacement between the pile and the soils. Therefore concrete is crushed in the middle part of the piles, as a result the structural model move and settle suddenly.

4. CONCLUSION

Failure mechanism of pile foundations during soil liquefaction have been investigated based on large-scale shaking table tests. The following conclusion are drawn;

(1)The damage to the pile heads occurred not only by the lateral force from the superstructure but also by the deformation of the surface layer, just before surface layer liquefied.
(2)The damage to the middle part of the piles was caused by the deformation of surface layer, immediately after the surface layer liquefied.

These test results suggest that the failure of the piles were controlled strongly by the process of liquefaction of the sand layer.

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conventional equivalent-linear calculations of ground response predict much greater depletion of high frequencies in the Fourier amplitude spectrum than found for observed data or for calculations by fully nonlinear methods. Consideration of the physics of the process leads to the conclusion that damping and perhaps modulus should be frequency-dependent, as has been suggested by Sugito (1995). Many years ago Dobry *et al.* (1971) showed that, for the standard hysteretic soil model, the motion produced by a small-amplitude cyclic load superimposed on a large load suffers less damping than the motion produced by the large load. This point is illustrated in Figure 1. Strictly speaking, the effects of the two different loads can not be superimposed if the response is nonlinear, but the approximate validity of superposition must be assumed if the equivalent-linear method is to be used at all. The spectrum of strain amplitude is similar to the spectrum of particle velocity, which decreases above the corner frequency of the earthquake approximately as the reciprocal of frequency. The spectrum of strain amplitude will therefore decrease with frequency above the corner frequency, and the strain-dependent damping used for equivalent-linear calculations should decrease with frequency above the corner frequency, rather than be constant over frequency as in the conventional equivalent-linear calculations.

Sugito (1995) has recently suggested that frequency-dependent modulus and damping be used in equivalent-linear calculations. He obtained a much better fit to Fourier amplitude spectral ratios from a downhole array in Japan using an equivalent-linear method with frequency-dependent modulus and damping than by the conventional equivalent-linear method. In this paper I compare soil response calculated by the conventional equivalent-linear procedure and by an equivalent-linear procedure featuring frequency-dependent damping with response calculated by a fully nonlinear procedure

It is not my intention to promote the use of equivalent linear methods for determining site response. The alternatives, fully nonlinear methods and empirical analysis of strong-motion data from seismically active areas, may be preferable, particularly the latter. In view of the popularity of equivalent-linear methods, however, I thought it useful to explore ways of improving them.

2. EQUIVALENT-LINEAR GROUND-RESPONSE CALCULATIONS WITH FREQUENCY-DEPENDENT DAMPING

In this section I compare nonlinear calculations using the method of Joyner and Chen (1975), which assumes a hysteretic soil model obeying the Masing rules (Ishihara, 1996), with the conventional equivalent-linear method that assumes modulus and damping independent of frequency and with a modified equivalent-linear method featuring frequency-dependent damping. For the modified method, in each soil layer, damping is independent of frequency below the predominant frequency f_0 defined in terms of the number of zero crossings. The value of f_0 (Newland, 1975) is given by

$$f_0 = \frac{1}{2\pi}\left(\frac{m_2}{m_0}\right)^{\frac{1}{2}},$$

where

$$m_k = \frac{1}{\pi}\int_0^{\infty} \omega^{\kappa} \, | E(f) |^2 \, d\omega,$$

$E(f)$ is the Fourier spectrum of strain, and $\omega = 2\pi f$. The damping for frequencies below f_0 is taken from the curve of damping versus strain. Above f_0 the damping is reduced by the factor (f_0/f). The soil profile is taken from the Gilroy #2 station, which recorded the Loma Prieta, California, earthquake. The shear velocities in the soil are given in Table 1, and the shear velocity in the underlying rock is 2.0 km/sec (Joyner *et al.*, 1981). The soil density is 2.0 gm/cm^3 and the rock density is 2.6 gm/cm^3. Below an overconsolidated zone 5m thick, the dynamic soil strength τ_{max} is given by

$$\tau_{\max} = C_s P_{ve},$$

where the strength coefficient C_s is equal to 1.0 and P_{ve} is the vertical effective stress. Above 5 m the preconsolidation vertical effective stress is assumed equal to the vertical effective stress at 5 m, and τ_{max} is given by

$$\tau_{\max} = C_s P_{ve} (OCR)^T,$$

where C_s is equal to 1.0, OCR is the overconsolidation ratio in terms of vertical effective stress, and T is equal to 0.75 (Ladd and Edgers (1972). The soil is presumed saturated below 20 m. No degradation is permitted and no pore-water diffusion.

The curves of modulus reduction and damping versus strain for both the conventional and modified equivalent linear methods are chosen to fit exactly the modulus and damping of the model used in the nonlinear calculations. By that choice we see how well the equivalent-linear methods represent the Masing-type hysteretic soil model, which is the most widely accepted model for the dynamic behavior of soils. The frequency-dependence of damping in the modified equivalent-linear method is chosen in an attempt to improve agreement with the nonlinear model. For both the conventional and modified equivalent-linear calculations, the peak strain in each layer is multiplied by the magnitude-dependent reduction factor recommended by Idriss and Sun (1992) before being used to determine modulus and damping.

The input motion considered is the north component recorded in the 1989 Loma Prieta, California, earthquake at Gilroy #1, a rock site 2 km from Gilroy #2. The peak acceleration is 0.44 *g*. The results are shown in Figure 2, where the pseudoacceleration response obtained by the nonlinear method is shown by the line, the response obtained by the conventional equivalent-linear method is shown by the plus signs, and the response obtained by the modified equivalent linear method is shown by the octagons. For periods between about 0.07 and 0.2 second the response obtained by the conventional equivalent-linear method is significantly less than the response obtained by the nonlinear method. The modified equivalent-linear method agrees much better with the nonlinear method, particularly for periods between 0.08 and 0.2 sec.

Both the conventional and modified equivalent-linear calculations illustrated in Figure 2 were made assuming that the damping goes to zero at zero strain. Measurements of soil-damping *in situ* (Gibbs *et al.*, 1994, Gibbs, written comm., 1998) give values of low-strain damping that are typically greater than 2 percent. Figure 3 shows the results of repeating the equivalent-linear calculations of Figure 2 with a low-strain damping of 2 percent. The nonlinear calculations are not repeated because the computer program used does not allow nonzero damping at low strain. The difference between conventional and modified equivalent-linear calculations is much less in Figure 3, and suggests that conventional equivalent-linear calculations may be adequate for many engineering purposes. The comparison in Figure 3 is based on response spectra. Larger differences would probably be seen in Fourier spectra. The results of Durward *et al.* (1996) and Sugito (1995) indicating that conventional equivalent-linear calculations underestimate high-frequency response were obtained by comparing Fourier spectra. Response spectra, however, are a better measure of engineering significance.

3. REFERENCES

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Table 1. Shear-Wave Velocity Profile at the Gilroy #2 Station (Joyner *et al.*, 1981)

Thickness (m)	Shear-Wave Velocity (m/sec)
2.5	160
5.0	274
12.5	400
20.0	320
60.0	630
20.0	440
60.0	650

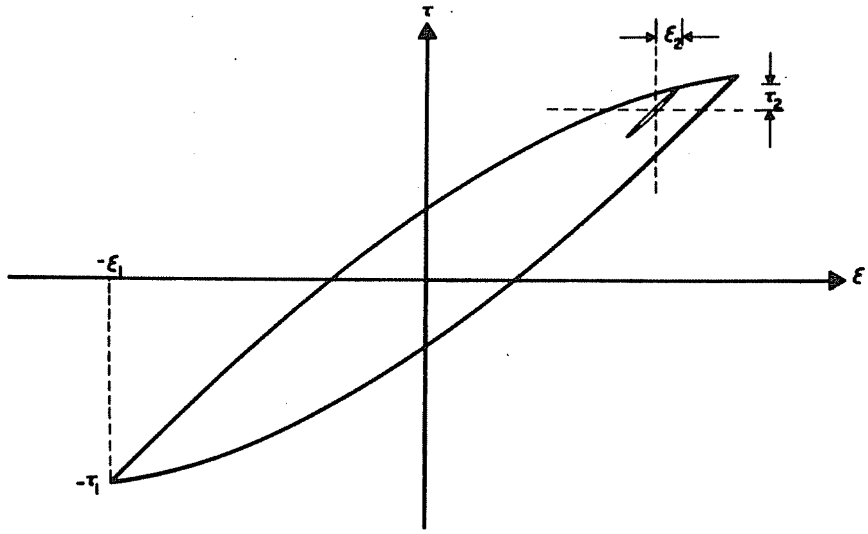

Figure 1.Idealized plot of stress (τ) versus strain (ε) for a soil sample showing the superposition of two sinusoidal loads. The energy dissipated in each loop is proportional to the area of the loop. (Dobry *et al.*, 1971)

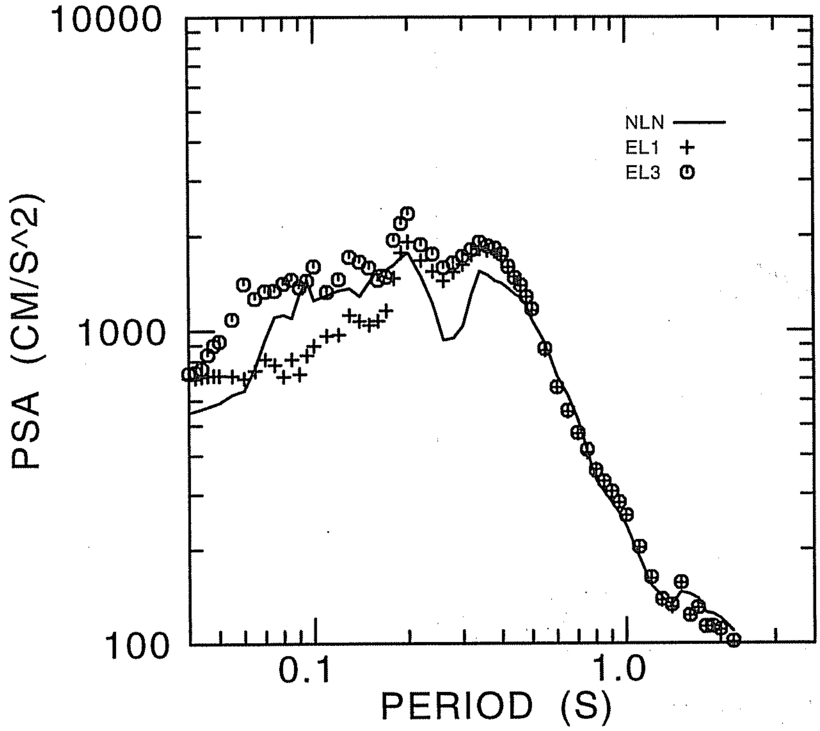

Figure 2. Pseudoacceleration response spectra (5 percent damping) for Gilroy station #2 computed by the nonlinear method (NLN), the conventional equivalent-linear method (EL1), and the modified equivalent-linear method described in the text (EL3). The input motion is the north component recorded in the 1989 Loma Prieta, California, earthquake at Gilroy station #1, a rock site 2 km from Gilroy # 2.

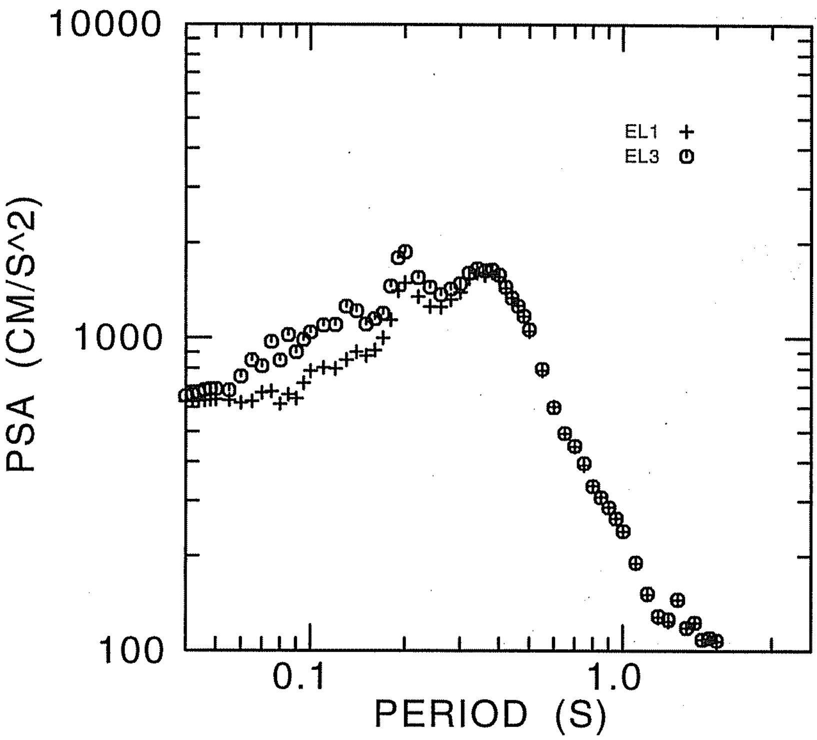

Figure 3. Pseudoacceleration response spectra from Figure 2 recomputed by the conventional equivalent-linear method (EL1) and the modified equivalent-linear method (EL3), assuming 2 percent low-strain damping.