I: State of the Art Talks

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Observations versus Analyses: Lateral Earth Pressures on an Embedded Foundation during Earthquakes and Forced Vibration Tests

Michio Iguchi, a) and Chikahiro Minowa b)

The characteristics of earth pressures during forced vibration tests and earthquakes are studied on the basis of observations recorded on both sides of a large-scale shaking table foundation. It is found that the earth pressures on both sides of the foundation during earthquakes tend to be induced in phase for lower frequencies contained in the surface ground motions, and out-of phase for higher frequencies. It is stressed that the in-phase phenomenon of the earth pressures cannot be explained by a conventional assumption of vertical incidence of seismic waves. It is also revealed that the earth pressures are induced in relation to the horizontal velocities of the foundation for rather lower frequencies, and to acceleration response for higher frequencies. These observations can well be simulated by a simplified lumped-mass model connected in series equivalently substituted for the lateral layered soil of the foundation.

INTRODUCTION

Transmission of ground motions into a structure and resistance of the supporting soil during vibration of a superstructure are soil-structure interaction phenomena accompanying with generation of dynamic earth pressures on the foundation. The observation of earth pressures during earthquakes, therefore, would play a key role in understanding of the transmission mechanism of ground motions as well as resistance mechanism of the surrounding soil for a superstructure.

The investigations of earth pressures during earthquakes have been conducted so far on the basis of theoretical, observational and experimental points of view. Above all, observations of earth pressures for actual structures or with use of model structures embedded in an actual soil have been providing valuable data in clarifying the generation mechanism of earth pressures on embedded foundations. Based on these observations, the frequency characteristics of earth pressures, the distribution of pressures on lateral sides of embedded foundations, phase characteristics of earth pressures induced on two opposite sides of the embedded foundation have been extensively studied. A detailed review about

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measurements of earth pressures during earthquakes has been presented by Ostadan and White (1998).

It is worthwhile noticing that following distinctive features on the earth pressures have been presented and discussed on the basis of earthquake observations;

(1) The observations include such that earth pressure on the opposite sides of the embedded foundation during earthquakes were induced in-phase, i.e. pull-and-pull or push-and-push phenomena were observed (Sakai et al. 1996, Uchiyama et al. 1999, Minowa et al. 2001). It is obvious that the fact cannot be explained by a conventional assumption of a vertical incidence of seismic waves.

(2) As for the earth pressures examined in relation to responses of foundation, there have been presented some observations showing different tendencies. One is that the earth pressures on the sides of embedded foundations are strongly related to the velocity motions of the foundation (Matsumoto et al. 1990, Sakai et al. 1996), the others are with acceleration motions of the foundation (Onimaru et al. 1994). There is also an observation showing that the earth pressure is caused by the relative motion between the structure and the surrounding soil (Kazama et al. 1988, Ostadan et al. 1998).

It should be noted that these phenomena are neither well documented nor explained from theoretical point of view. Some simplified models to predict the earth pressures on the rigid wall during earthquakes have been presented (Scott 1973, Veletsos et al. 1994a, Veletsos et al. 1994b). Unfortunately, these models fail to explain above described phenomena.

The observations of earth pressures induced by earthquake ground motions have been conducted on both sides of a large-scale shaking table foundation in Tsukuba. Simultaneous observations of free-field ground motions as well as the response of the foundation have been made (Iguchi et al. 2000, Iguchi et al. 2001). The dense earthquake observations around or on the shaking table foundation permit us to study the characteristics of earth pressures in relation to the ground motions and to the response of foundation as well. Forced vibration tests have been also performed for the foundation to obtain the earth pressures during the excitations.

The objective of this paper is to analyze the records of the earth pressures observed on the lateral sides of the foundation during earthquakes and the forced vibration tests as well. Special emphasis is placed to extract above described phenomena on the basis of the observed data. A simplified analysis model of a lumped-mass system connected in series is also presented to simulate numerically the observations. The focus of this paper is to show and discuss how extent could we explain the observations with use of the simplified model.

FOUNDATION AND EARTH PRESSURE MEASUREMENT

Figure 1 shows a large-scale shaking table foundation in National Research Institute for Earth Science and Disaster Prevention in Tsukuba. The size of the foundation is \(25 \times 39\) m in plane and the base of the foundation is directly supported on firm sand at a depth of 8.2 m. The section of the foundation and soil profile are shown in figures 1(a) and 2. The weight of the foundation and the shaking table is about 11,600tf (113.7MN) and 180tf (1.76MN), respectively. The total weight is approximately amount to 20,000tf including the weight of steel-made super-structure and corresponds almost to the excavated soil of the foundation. The fundamental frequency of the soil-foundation system is about 4.1 Hz in the x-direction.
that was observed by the forced vibration test. It has been confirmed that the foundation behaves as a rigid body within frequencies less than 10 Hz. The soil constants at the site were measured to a depth of about 40m as shown in figure 2, and more details may be found elsewhere (Minowa et al. 1991).

Figure 1(b) shows the location of earth pressure gauges deployed on the sides of the foundation. On each side of the foundation, five earth pressure gauges have been instrumented at different depths, but one installed on the west side was out of function and is omitted from the figure. The simultaneous observations of earthquake ground motions have been conducted at depths of 1m and 40m below the soil surface. We refer the ground motions recorded at the depth 1m to as surface ground motions. Regarding the foundation, the earthquake responses of the foundation have been observed at several points with accelerographs and a velocity seismograph as shown in figure 1(a). The horizontal response of the foundation is represented by seismograms recorded at the point S3, which is located almost at the center of the foundation.

**Earth pressure during forced vibration tests**

The earth pressures have been measured during the step-sweep forced vibration tests. A harmonic excitation force was generated by driving a shaking table in the frequency range from 1 to 20 Hz with a frequency step 0.2 Hz. The horizontal excitation was applied at 1m below the foundation surface. Two levels of excitation forces were performed in the series of experiments: one was the test conducted by driving the shaking table so as to generate the acceleration of about 100 gals on the shaking table, which is equivalent to about 20tf of excitations, and the other was the test of about 500 gals throughout the frequencies.

There was no evidence of nonlinear phenomena such as separation of the lateral soil from the foundation as far as being judged by inspection of the recorded waveforms of the earth pressures.
Figures 3 and 4 show the frequency characteristics of earth pressures observed on both sides of the foundation. The results shown in figure 3 are the amplitude and phase characteristics of earth pressures normalized by unit horizontal displacement of the foundation. The results of earth pressures shown in figure 3 indicate that for lower frequencies less than 3–4 Hz the amplitudes of the earth pressure are almost constant and tends to increase for higher frequencies. As for the phase characteristics, it will be noticed...
that the earth pressures are induced in-phase to the displacement motion of the foundation. This fact indicates that the seismic deformation method, which is widely used in the seismic response analysis of underground structures, may not be valid for higher frequencies. Similarly, figure 4 shows the characteristics of earth pressures normalized by unit horizontal velocity of the foundation. An inspection of these results shown in figure 4 reveals that the magnitudes of earth pressure varies with frequencies but the phases tend to be almost constant in higher frequencies more than $3 \sim 4$ Hz. This indicates that the earth pressures are induced in phase to the velocity response of the foundation in the frequency range more than $3 \sim 4$ Hz.

The above mentioned frequencies $3 \sim 4$ Hz may be presumed as the fundamental frequency of two layering surface soil. The fundamental frequency can be approximated by means of a quarter-wave-length method, and will be given as follows with the soil constants shown in figure 2.

\[
f_1 = \frac{1}{4} \left(\frac{H_1}{V_{s1}} + \frac{H_2}{V_{s2}}\right)^{-1} = 3.5 \text{Hz}
\]

where $H_1$ and $H_2$ are the thickness of the first and the second layers of surface soil, and $V_{s1}$ and $V_{s2}$ are corresponding shear wave velocities. Thus, 3.5Hz may be considered to be the fundamental frequency of the two layered surface soil.

**Earth pressure observation during earthquakes**

The observations of earth pressures induced by earthquake ground motions have been conducted for six years from 1991 to 1996 and the records of about 30 earthquakes had been obtained. The details of the earthquake records may be found elsewhere (Iguchi et al. 2000, Iguchi et al. 2001). In order to discuss the characteristics of the earth pressures in relation to frequency component contained in the earthquake ground motions on the soil surface, the recorded earthquake motions was categorized into three groups (groups A, B and C). The grouping was made according to the predominant frequency components included in the earthquake acceleration motions (NS component) recorded on the soil surface; the earthquake ground motions including predominantly the lower frequencies less than 1Hz were categorized into group A; the earthquake motions with higher components (more than about 3 to 4Hz) were grouped into C, and group B was characterized by the motions having intermediate frequency components between groups A and C. Table 1 shows the earthquake parameters of the three records selected as the representative motions picked up from each group. Figure 5 shows the normalized Fourier spectra of the representative motions chosen from the respective groups. The spectra are smoothed by using the Parzen window with a bandwidth of 0.1Hz.

<table>
<thead>
<tr>
<th>Eq. No</th>
<th>Date</th>
<th>Epicenter</th>
<th>Depth (Km)</th>
<th>Magnitude</th>
<th>PGA (gal)</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1991 Oct 19</td>
<td>N36.08, E139.92</td>
<td>59</td>
<td>4.3</td>
<td>43.73</td>
<td>C</td>
</tr>
<tr>
<td>17</td>
<td>1993 Oct 12</td>
<td>N32.02, E138.24</td>
<td>390</td>
<td>7.0</td>
<td>27.15</td>
<td>B</td>
</tr>
<tr>
<td>30</td>
<td>1996 Sep 11</td>
<td>N35.07, E141.03</td>
<td>30</td>
<td>6.6</td>
<td>26.65</td>
<td>A</td>
</tr>
</tbody>
</table>
Phase characteristics of earth pressures induced on both sides of the foundation can be extracted by calculating the motion products of the earth pressures recorded on both sides. The computed results of the motion products are shown in figures 6(a), (b) and (c) for the representative motions of each group. The positive value of the product can be interpreted as the earth pressures on both sides being induced in phase on both sides, and the negative values of the product, on the other hand, imply the generation of earth pressures with a phase lag out of 180 degrees on two sides. Inspection of the results shown in figure 6(a) indicates that for the ground motions of group A the earth pressures induced on both sides are almost in phase throughout the record. It should be noticed that the in-phase phenomenon of the earth pressures cannot be explained by the conventional assumption of the vertical incidence of seismic waves.

As for group B shown in figure 6(b), the earth pressures induced on both sides of the foundation are showing out-of-phase in the primary portion of the time history and tend to shift to in-phase with progress of time. With regard to the motions of group C, which contains higher frequencies, out-of-phase and in-phase phenomena are recognized alternately throughout the whole time history as shown in figure 6(c). Summarizing the results shown in figures 6(a), (b) and (c), it may be noted that the phase characteristics of earth pressures induced on both sides of the foundation are strongly affected by frequency components contained in the ground motions and tend to be induced in phase for the ground motions with lower frequencies.

Another point to be discussed is what components of foundation motions are closely related to the earth pressures. Observed time histories of earth pressures and velocity response of foundation are shown simultaneously in figures 7(a), (b) and (c) for respective ground motions of groups A, B and C. It will be noticed that the earth pressures are closely correlated with velocity motions of foundation for groups A and B. As for group C, on the other hand, the close correspondence between the two is not recognized as shown in figure 7(c). Figure 8 shows time histories of earth pressures and acceleration motions of the foundation from 0 to 10 sec (top) and 10 to 20 sec (bottom). A strong correlation between the earth pressures and acceleration motions of the foundation for group C is recognized. Finally, figures 9(a) to (e) show Fourier amplitude spectrum ratios of the earth pressures observed at different points to the velocity motions of foundation. The results are smoothed by use of Parzen window with a bandwidth of 0.2 Hz. It will be noticed that the spectrum ratios are almost constant for frequencies less than $4 \sim 5$ Hz and tend to increase almost linearly for higher frequency. This fact indicates that earth pressures are induced in accordance with
velocity motions of the foundation for lower frequencies and to acceleration motions for higher frequencies.

Figure 6. Time histories and motion products of earth pressures induced on opposite sides of the foundation during earthquakes.
Figure 7. Time histories of earth pressures and velocity responses of foundation.

(a) Group A

(b) Group B

(c) Group C

Figure 8. Time histories of earth pressure and acceleration response of foundation (Group C).

Figure 9. Spectrum ratios of earth pressures to velocity response of foundation.
ANALYSIS OF LATERAL SOIL RESISTANCE

Description of problem and formulation

The objective of this chapter is to present formulation for analyses of the lateral soil resistance to the foundation when excited by a lateral harmonic force applied on the foundation. The foundation-soil system considered in this study is shown in figure 10. The laterally extending soil is composed of two horizontal layers with hysteretic material damping, which is supposed to be supported rigidly at the base. The soil is assumed to be composed of two-dimensional medium of plane strain with unit width. The coordinate systems $x$ and $z$ are chosen as shown in figure 11.

On the assumption that the vertical displacements in each layer are negligibly small compared to the horizontal component, the equations of harmonic motion for each layer can be written by,

Layer 1: \( (1 + 2ih_1) \left\{ E_1 \frac{\partial^2 u_1}{\partial x^2} + G_1 \frac{\partial^2 u_1}{\partial z^2} \right\} + \rho_1 \omega^2 u_1 = 0 \) \hspace{1cm} (1)

Layer 2: \( (1 + 2ih_2) \left\{ E_2 \frac{\partial^2 u_2}{\partial x^2} + G_2 \frac{\partial^2 u_2}{\partial z^2} \right\} + \rho_2 \omega^2 u_2 = 0 \) \hspace{1cm} (2)

where $\omega =$circular frequency of harmonic excitation, $u_i =$ horizontal displacement for $l$-th layer ($l = 1, 2$), $\rho_i =$ mass density of medium, and $h_i =$ material damping factor of hysteretic type, and $E_i =$ Young’s modulus of soil, which can be expressed with Lame’s constants $G_i, \lambda_i$ as follow.

\[ E_i = \lambda_i + 2G_i, \quad l = 1, 2 \] \hspace{1cm} (3)

Figure 10. Analysis model of foundation-lateral soil system.

Figure 11. Coordinate system and earth pressure induced on the interface between foundation and the lateral soil.
Under the condition of absence of vertical displacements, the normal stress in the soil \( \sigma_x(x,z) \) may be expressed with horizontal displacement as follows.

\[
\sigma_x(x,z) = (1 + 2i h_i) E_i \frac{\partial u_i}{\partial x}, \quad l = 1, 2
\]

The boundary conditions on the surface, on the intermediate soil interface and at the bottom of soil may be expressed by,

\[
z = H_1 + H_2 \quad ; \quad \frac{du_1}{dz} \bigg|_{z=H_1+H_2} = 0 \quad (5-1)
\]

\[
z = H_2 \quad ; \quad u_1(H_2) = u_2(H_2) \quad (5-2)
\]

\[
(1 + 2i h_1) G_1 \frac{du_1}{dz} \bigg|_{z=H_2} = (1 + 2i h_2) G_2 \frac{du_2}{dz} \bigg|_{z=H_2} \quad (5-3)
\]

\[
z = 0 \quad ; \quad u_2(0) = 0 \quad (5-4)
\]

where \( H_1 \) and \( H_2 \) are the thickness of the first and the second layer, respectively.

In a similar manner, the boundary conditions on the lateral surface of the foundation and the radiation condition at infinity may be expressed by

\[
x = 0 \quad ; \quad (1 + 2i h_1) E_1 \frac{\partial u_1}{\partial x} = -p(z), \quad H_2 \leq z \leq H_1 + H_2 \quad (6-1)
\]

\[
(1 + 2i h_2) E_2 \frac{\partial u_2}{\partial x} = -p(z), \quad 0 \leq z \leq H_2 \quad (6-2)
\]

\[
x = \infty \quad ; \quad u_1 \rightarrow 0, \quad u_2 \rightarrow 0 \quad (6-3)
\]

where \( p(z) \) is the amplitude of lateral normal stress that the foundation exerts on the lateral surface of the layered soil.

In analyzing of Eqs (1) and (2) under the boundary conditions above, we summarize these two equations as follows for convenience.

\[
E^*(z) \frac{\partial^2 u}{\partial x^2} + \frac{d}{dz} \left( G^*(z) \frac{du}{dz} \right) + \rho(z) \omega^2 u = 0 \quad (7)
\]

where,

\[
u(x,z) = \begin{cases} \nu_1(x,z) & ; \quad H_2 < z < H_1 + H_2 \\
\nu_2(x,z) & ; \quad 0 < z < H_2 
\end{cases} \quad (8)
\]

\[
G^*(z) = \begin{cases} (1 + 2i h_1) G_1 & ; \quad H_1 < z < H_1 + H_2 \\
(1 + 2i h_1) G_2 & ; \quad 0 < z < H_2 
\end{cases} \quad (9-1)
\]

\[
E^*(z) = \begin{cases} (1 + 2i h_1) E_1 & ; \quad H_2 < z < H_1 + H_2 \\
(1 + 2i h_1) E_2 & ; \quad 0 < z < H_2 
\end{cases} \quad (9-2)
\]

and,
Employing the method of separation of variables for the displacement \( u(x,z) \), we write as

\[
\begin{align*}
  u(x,z) &= \begin{bmatrix} u_1(x,z) \\ u_2(x,z) \end{bmatrix} = X(x)U(z) \\
  \rho(z) &= \begin{cases} 
    \rho_1(z) & ; H_2 < z < H_1 + H_2 \\
    \rho_2(z) & ; 0 < z < H_2 
  \end{cases} 
\end{align*}
\]  

(9-3)

As for the displacement component along the depth of soil \( U(z) \), we adopt the fundamental mode shape of the two layering soil. Letting \( U(z) \) be as

\[
U(z) = \begin{cases} 
  U_1(z), & H_2 \leq z \leq H_1 + H_2 \\
  U_2(z), & 0 \leq z \leq H_2 
\end{cases}
\]

(11)

the mode shapes for the first and second layers \( U_1(z), U_2(z) \) may be given by

\[
\begin{align*}
  U_1(z) &= C \cos \left( \frac{\omega}{V_{s1}} (H_1 + H_2 - z) \right), \quad H_2 \leq z \leq H_1 + H_2 \\
  U_2(z) &= C \left( \frac{\cos \left( \frac{\omega}{V_{s1}} H_1 \right)}{\cos \left( \frac{\omega}{V_{s2}} H_2 \right)} \sin \left( \frac{\omega}{V_{s2}} z \right) \right), \quad 0 \leq z \leq H_2
\end{align*}
\]

(12-1, 12-2)

where \( C = \text{integral constant} \), and \( V_{s1}, V_{s2} \) are shear wave velocities of soil layers as given by

\[
V_{s1}^2 = \frac{G_1}{\rho_1}, \quad V_{s2}^2 = \frac{G_2}{\rho_2}
\]

(13)

It may be easily shown that these functions satisfy the boundary conditions of Eqs (5). In Eqs (12) \( \omega_1 \) is the fundamental circular frequency of the shear-column of two layering soil without damping, and is given by the minimum root of the following characteristic equation.

\[
-\frac{G_2}{V_{s2}^2} \cos \omega \frac{V_{s1}}{V_{s2}} H_2 + \frac{G_1}{V_{s1}^2} \sin \omega \frac{V_{s1}}{V_{s2}} H_1 \sin \omega H_2 = 0
\]

(14)

The participation factor of the shear-column when vibrating with the fundamental mode may be given by

\[
\beta_1 = \frac{\int_{0}^{H_1+H_2} \rho(z) \cdot U(z) dz}{\int_{0}^{H_1+H_2} \rho(z) [U(z)]^2 dz}
\]

(15)

Next, substitution from Eq. (10) into Eq. (7) gives

\[
E^* \frac{d^2 X}{dz^2} U + X \frac{d}{dz} \left( G^*(z) \frac{dU}{dz} \right) + \rho(z) \omega^2 U \cdot X = 0
\]

(16)

Multiplying Eq. (16) by \( U(z) \) and integration with respect to \( z \) in the range \( [0, H_1 + H_2] \), and multiplying \( \beta_1^2 \) on both sides of the equation, then we will arrive at
\[ n k^e \frac{d^2 X}{dx^2} - k^e X + m^e \omega^2 X = 0 \]  \hspace{1cm} (17)

This equation of motion is equivalent to that of a system which is composed of an axial rod supported on continuously distributed shear soil spring. The parameters appearing in this equation \( m^e \), \( s k^e \) and \( n k^e \) are given by

\[
m^e = \beta^2 \int_0^{H_1+H_2} \rho(z) \{U(z)\}^2 \, dz \]  \hspace{1cm} (18-1)

\[
s k^e = -\beta^2 \int_0^{H_1+H_2} \frac{d}{dz} \left( G^*(z) \frac{dU}{dz} \right) U(z) \, dz \]

\[
= \beta^2 \int_0^{H_1+H_2} G^*(z) \left( \frac{dU}{dz} \right)^2 \, dz \]  \hspace{1cm} (18-2)

\[
n k^e = \beta^2 \int_0^{H_1+H_2} E^*(z) \{U(z)\}^2 \, dz \]  \hspace{1cm} (18-3)

It should be noted that these are complex valued except for \( m^e \).

Substituting from Eq. (15) into Eqs (18-1) and (18-2), we will obtain other forms for \( m^e \), \( s k^e \) as

\[
m^e = \left[ \int_0^{H_1+H_2} \rho(z) \cdot U(z) \, dz \right]^2 \left[ \int_0^{H_1+H_2} \rho(z) \{U(z)\}^2 \, dz \right] \]  \hspace{1cm} (19)

\[
s k^e = \frac{\left[ G^*(z) \frac{dU}{dz} \right]_{z=0}^2}{\int_0^{H_1+H_2} G^*(z) \left( \frac{dU}{dz} \right)^2 \, dz} \]  \hspace{1cm} (20)

### Equivalent multi-lumped-mass model

Introducing a discretization procedure for Eq. (17) with an equally spaced finite difference approximation, it is possible to rewrite as

\[-n K^e X_{j-1} + \left( 2 \frac{n K^e}{\Delta x} + s K^e - M^e \omega^2 \right) X_j - n K^e X_{j+1} = 0 \]  \hspace{1cm} (21)

where \( X_j \) is the horizontal displacement of the j-th mass, and other parameters appearing in these equations are defined as

\[ n K^e = \frac{1}{\Delta x} n k^e, \quad s K^e = \Delta x k^e, \quad M^e = \Delta x m^e \]  \hspace{1cm} (22)

in which \( \Delta x \) is a mass interval of spacing. We will notice that Eq. (21) is nothing but an equation of motion of a lumped-mass system connected in series as shown in figure 12, and
the constants of the system are those given in Eq. (22). The equivalent mass height \( H^e \) of the system may be determined as the value of \( z \) that satisfies the following equation.

\[
\beta_1 \cdot U(z) = 1
\]  

(23)

The equivalent resultant force that the foundation exerts on the first end mass will be expressed as follow.

\[
P^e = \beta_1 \int_0^{H_H+z_H} p(z)U(z)dz
\]

(24)

If we assume that the lateral normal stress on the interface between the foundation and the lateral soil is uniformly distributed throughout the depth \( z \) \( (p(z) = p_0) \), then Eq. (24) reduces to

\[
P^e = p_0 \left[ \beta_1 \int_0^{H_H+z_H} U(z)dz \right]
\]

(25)

where \( p_0 \) is the amplitude of the normal stress.

A big advantage of the reduction into a lumped-mass system is that thus obtained discreet system permits us to extend to nonlinear analyses of the lateral soil.

**Figure 12. Multi-lumped-mass system.**

**Analysis of finite difference equation**

We try to solve the finite difference equation shown in Eq. (21). The closed form solution of this equation has been presented by Tajimi (1990). Following the Tajimi’s approach, we assume for the solution of Eq. (21) that satisfies the radiation condition of Eq. (6-3) as

\[
X_j = A e^{-\alpha j}, \quad j = 1, 2, \cdots
\]

(26)

where \( A \) is an arbitrary constant to be determined by the boundary condition at \( j = 1 \).

Substituting from Eq. (26) into Eq. (22) we obtain

\[
Ch \alpha = \frac{2_n K^e + \frac{1}{2} K^e - M^e \omega^2}{2_n K^e} = 1 + \frac{M^e}{2_n K^e} (\omega_i^2 - \omega^2)
\]

(27)

It should be noted that \( \alpha \) is a function of frequency and a complex value of \( \text{Re}(\alpha) \geq 0 \). In this equation a relation \( s K^e / M^e = s k^e / m^e = \omega_i^2 \), in which \( \omega_i \) indicates the fundamental
Next step is to determine the arbitrary constant $A$ in Eq. (26). The constant $A$ may be determined from the boundary conditions for the first mass. Taking into account that the governing region of the end mass is a half of the other masses, the equation of motion for the first mass ($j = 1$) may be given as follows.

\[
\left( s K^e + \frac{1}{2} s K^e - \frac{1}{2} M^e \omega^2 \right) X_1 - n K^e X_2 = P^e
\]

Substituting from Eq. (26) into Eq. (28), it may be found that $A$ is given by

\[
A = e^{\alpha} \frac{1}{Sh \alpha} P^e
\]

Substitution Eq. (29) into Eq. (26) leads to the solution of the finite difference equation as shown by

\[
X_j = \frac{P^e}{n K^e Sh \alpha} e^{-\alpha(j-1)}
\]

By setting $j = 1$ in this equation, we will obtain the relationships between the equivalent excitation force $P^e$ and the displacement of the first mass as shown by

\[
P^e = n K^e Sh \alpha \cdot X_1
\]

In this equation $n K^e Sh \alpha$ implies the equivalent dynamic stiffness of the lateral layering soil of a unit width perpendicular to the plane, in which $n K^e$ is the lateral stiffness of the lumped-mass system when the second mass kept immovable and $Sh \alpha$ represents the effects of the other masses connected laterally. Making use of the relation $Ch \cdot \alpha - Sh \cdot \alpha = 1$ and from Eq. (27), we will obtain an explicit form for $Sh \alpha$ as

\[
Sh \alpha = \sqrt{\frac{M^e}{n K^e (\tilde{\omega}_1^2 - \omega^2)}} \sqrt{1 + \frac{M^e}{4 n K^e (\tilde{\omega}_1^2 - \omega^2)}}
\]

Finally, substituting from Eq. (31) into Eq. (25), the magnitude of earth pressure $p_0$ generated on the side of the foundation may be evaluated by

\[
p_0 = \frac{K^e Sh \alpha}{\beta_1^{H_f + H_z} U(z)dz} X_1
\]

It should be noted that $X_1$ in Eq. (33) is equivalent to the amplitude of a horizontal displacement of the foundation at the equivalent height $H^e$, which may be evaluated on the basis of the observations during the forced vibration tests.
ANALYSIS OF EARTH PRESSURE DURING EARTHQUAKES

Analysis model and assumptions

The model adopted in this study for the analysis of earth pressures during earthquakes is illustrated in figure 13. It consists of a rigid foundation supported by sway and rocking springs and dashpots which represent radiation damping into subsoil below foundation, and two series of lumped-mass system representing the dynamic resistance of the laterally extending soil. As described earlier the observed results of the earth pressures induced on the opposite sides of the foundation during earthquakes cannot be explained by an assumption of vertical incidence of seismic waves. In the analysis of the soil-foundation system subjected to ground motions, therefore, we assume in this paper an oblique incidence of seismic waves that propagate in the x-direction.

In formulation of response analyses of the system, the whole system is divided into three substructures: two of them are right and left lateral lumped-mass systems as shown in figure 14, and the other is the rigid foundation subjected to the ground motions at the base and horizontal forces from the lateral soil. The input motions into the foundation from the lateral soil are included in the formulation of the lateral lumped-mass systems. The coordinate $x$ is defined as shown in figure 13 with the origin at the center of the foundation base. In the seismic response analysis of the whole system, it will become a key step to evaluate the response of lateral lumped-mass systems when subjected to both the ground motions and horizontal forces exerted from the lateral soil.

**Figure 13. Lumped-mass model for response analysis of foundation-lateral soil system.**

**Figure 14. (a) Lumped-mass system for the lateral soil of the right side, and (b) the left side.**
Earthquake response analyses of system

The response of the lumped-mass system connected in series shown in figure 14 may be expressed by the sum of two responses. One is the response to a harmonic excitation applied at the end mass, which has been given before, and the other is one to the harmonic ground motions. Thus, the total response of the system may be expressed by the sum of two responses as

\[ X_j = X_j^{(1)} + X_j^{(2)} \]  \hspace{1cm} (34)

where the first term of the right side represents the response to the excitation applied at the end mass, and the second to the ground motions. Making use of the result expressed in Eq. (30) the response to the excitation at the end mass of the right side shown in figure 14(b) may be expressed by

\[ X_j^{(1)} = \frac{P^e_R}{nK^eSh_\alpha} e^{-\alpha(j-1)} \]  \hspace{1cm} (36)

where \( P^e_R \) indicates the equivalent lateral force generated on the interface between the foundation and the lateral soil extending to the right direction.

Next step is to analyze the lumped-mass system when subjected to traveling ground motions at the base of each mass. A harmonic seismic waves traveling to x-direction with an apparent wave velocity c on the surface of the supporting bed rock, may be expressed as follows.

\[ u_g(x,t) = u_0 e^{i\omega(t-x/c)} \]  \hspace{1cm} (37)

where \( u_0 \) is the amplitude of the harmonic ground motions.

Letting the response of the \( j-th \) mass of the system be \( X_j^{(2)} e^{i\omega t} \), the equation of motion governing the lumped-mass system may be expressed by

\[ -s K^e X_j^{(2)} + \left( 2nK^e + s K^e - M^e \omega^2 \right) X_j^{(2)} - s K^e X_{j+1}^{(2)} = s K^e U_{g,j} \]  \hspace{1cm} (38)

where \( U_{g,j} \) is the amplitude of ground motion at the \( j-th \) mass, which is defined as follows with wave number \( k = \omega / c \):

\[ U_{g,j} = u_0 e^{-ikx_j} \]  \hspace{1cm} (39)

in which \( x_j \) is horizontal distance from the center of the foundation to the \( j-th \) mass, and may be given as,

\[ x_j = L / 2 + \Delta x(j-1) \]  \hspace{1cm} (40)

with full length of the foundation \( L \) and the spacing of mass \( \Delta x \).

For the first mass, the equation of motion may be expressed by

\[ \left( 2nK^e + s K^e - M^e \omega^2 \right) X_1^{(2)} - 2nK^e X_2^{(2)} = s K^e U_{g1} \]  \hspace{1cm} (41)

The general solution for Eq. (38) will be given as follows.
\[ X_j^{(2)} = H_R(\omega) \left[ A e^{-\alpha j} + e^{-i(kL/2 + \Delta x(j-1))} \right] u_0 \]  

where \( A \) is an arbitrary constant to be decided so as to satisfy Eq. (41), and is given as

\[ A = -i \frac{\sin k\Delta x}{S\alpha} e^{i(\alpha-\Delta L)/2} \]  

In Eq. (42), \( H_R \) is defined as

\[ H_R(\omega) = \frac{\tilde{\omega}_1^2}{\tilde{\omega}_1^2 - \omega^2 + \frac{2\pi K^e}{M^e} (1 - \cos k\Delta x)} \]  

where \( \tilde{\omega}_1^2 \) indicates the fundamental circular frequency of the lateral layering soil and defined as

\[ \tilde{\omega}_1^2 = \frac{s K^e}{M^e} \]  

The function \( H_R \) is interpreted as a frequency response factor of the multi-lumped-mass system when subjected to non-vertical incidence of seismic waves.

Thus, the total harmonic response of the first mass in the right side when subjected to both the horizontal excitation from the foundation and the ground motions, will be obtained by substitution of Eqs (36) and (42) with Eq. (43) into Eq. (34), and it leads to

\[ X_1 = H_R(\omega) e^{-i(kL/2)} \left[ 1 - i \frac{\sin k\Delta x}{S\alpha} \right] u_0 + \frac{1}{n K^e S\alpha} P_R^e \]  

In a similar manner, the harmonic response of the first mass in the left side of the foundation \( X_{-1} e^{i\omega t} \) will be finally expressed by

\[ X_{-1} = H_L(\omega) e^{i(kL/2)} \left[ 1 + i \frac{\sin k\Delta x}{S\alpha} \right] u_0 - \frac{1}{n K^e S\alpha} P_L^e \]  

where \( P_L^e \) is an amplitude of the horizontal excitation applied at the first mass in the left side. The \( H_L \) in Eq. (47) is the same as \( H_R \), which is defined in Eq. (44).

It should be noted here that the responses of the first masses in the right and left sides of the foundation, \( X_1 \) and \( X_{-1} \), are equal to the foundation response of the foundation at the level of the equivalent height of the mass system.

### Formulation for response analysis of foundation

The final step of the analyses is the analysis of response of the rigid foundation subjected to excitations from the both sides of the foundation and earthquake ground motions at the base. In evaluation of the foundation input motion when subjected to oblique incident seismic waves, an averaging procedure proposed by Iguchi (1973) is adopted. The horizontal component of the foundation input motion \( U^e e^{i\omega t} \) may be evaluated by

\[ U^e = \frac{\sum_{j=1}^{N} X_j^{(2)} e^{i(j-1)(kL/2 + \Delta x)}}{\sum_{j=1}^{N} e^{i(j-1)(kL/2 + \Delta x)}} \]
\[ U^* e^{j \omega t} = \frac{1}{A} \int_A u_g(x,t) dA \]  

where \( A \) is base area of the foundation. Substitution from Eq. (37) into Eq. (48) and integration over the area leads to

\[ U^* = \frac{s i n(kL/2)}{kL/2} u_0 \]  

Assuming vertical incidence of incoming waves \( (c \rightarrow \infty \) or \( k \rightarrow 0 \)) in Eq. (49), the amplitude of the foundation input motion will be equal to that of the incident wave and thus \( U^* = u_0 \).

The equation of motion for the rigid foundation subjected to both horizontal excitations from the lateral sides of the foundation and the ground motions may be expressed as follows.

\[ \left[ \begin{array}{c} K_H \\
\cdot \\
K_R / (H^c)^2 \end{array} \right] - \omega^2 \left[ \begin{array}{c} M_0 \\
\cdot \\
h_G M_0 / H^c \\
J / (H^c)^2 \end{array} \right] \left\{ \begin{array}{c} \Delta_0 \\
\cdot \end{array} \right\} = \omega^2 M_0 \left\{ \begin{array}{c} 1 \\
\cdot \\
h_G / H^c \end{array} \right\} U^* - B \left[ \begin{array}{c} 1 \\
\cdot \end{array} \right] \left\{ \begin{array}{c} -P^e_R \\
\cdot \end{array} \right\} \]  

where \( \Delta_0 \) and \( \Phi_0 \) are horizontal and rocking motions at the base of the foundation, \( h_G \) = the height of the gravity center of the foundation from the base, \( B = \) width of the foundation, \( H^c = \) equivalent height of mass connected in series, \( M_0 = \) mass of the foundation, and \( J = \) a mass moment of inertia of the foundation with respect to its bottom, which may be evaluated by

\[ J = J_0 + h_G^2 M_0 \]  

with a mass moment of inertia \( J_0 \) with respect to the gravity center of the foundation.

In Eqs (46) and (47), it should be reminded that \( X_1 \) and \( X_{-1} \) are equal to the horizontal displacements of the foundation at the equivalent height of the mass system. Thus we hold

\[ X_1 = X_{-1} = \Delta_0 + H^c \Phi_0 - U^* \]  

Substituting from Eq. (52) into Eqs (46) and (47), we will obtain the expressions for the resultant earth pressures on the lateral sides of the foundation \( P^e_R \) and \( P^e_L \).

\[ P^e_R = n K^c Sh \alpha (\Delta_0 + H^c \Phi_0 + U^*) - H(\omega) n K^c e^{-ikL/2} (Sh \alpha - i \sin k \Delta x) u_0 \]  

\[ P^e_L = -n K^c Sh \alpha (\Delta_0 + H^c \Phi_0 + U^*) + H(\omega) n K^c e^{ikL/2} (Sh \alpha + i \sin k \Delta x) u_0 \]  

where we set \( H(\omega) = H_R(\omega) = H_L(\omega) \).

Substituting from Eqs. (53) and (54) into Eq. (50), the equation of motion for the foundation may be finally expressed by

\[ \left[ \begin{array}{ccc} K_H + 2K_L & 2K_L \\
2K_L & K_R / (H^c)^2 + 2K_L \end{array} \right] - \omega^2 \left[ \begin{array}{cc} M_0 & h_G M_0 / H^c \\
h_G M_0 / H^c & J / (H^c)^2 \end{array} \right] \left\{ \begin{array}{c} \Delta_0 \\
\cdot \end{array} \right\} = \omega^2 M_0 \left\{ \begin{array}{c} 1 \\
h_G / H^c \end{array} \right\} U^* - B \left[ \begin{array}{c} 1 \\
\cdot \end{array} \right] \left\{ \begin{array}{c} -P^e_R \\
\cdot \end{array} \right\} \]
\[ e^{2}M_{0} \left\{ \frac{1}{h_{e}/H} \right\} U^{*} - 2K_{L} \left\{ \frac{1}{1} \right\} U^{*} + 2H(\omega)K_{L} \left( \cos kL/2 - \frac{\sin kL/2 \sin k\Delta x}{Sh\alpha} \right) \left\{ \frac{1}{1} \right\} u_{0} \]  

(55)

where \( K_{H} \) and \( K_{R} \) are the horizontal and rocking springs defined at the base of the foundation, which are the function of frequencies and complex valued. These complex springs represent the soil stiffness evaluated excluding the effects of lateral soil. In addition, \( K_{L} \) in Eq. (55) is a complex spring for the lateral side of the foundation.

\[ K_{L} = B_{n}^{*}Sh\alpha \]  

(56)

Once the sway and rocking responses of the foundation are solved from Eq. (55) for given ground motions, the resultant earth pressures on the lateral sides of the foundation can be evaluated from Eqs (53) and (54). The earth pressure in a unit area under the assumption of uniform distribution of the contact stress may be evaluated from Eq. (33).

**Theoretical discussion of earth pressure during earthquakes**

Based on Eqs (53) and (54) the expressions of the lateral earth pressures on both sides of the foundation under the action of earthquake ground motions may be rewritten as follow.

\[ P_{n} = \frac{e^{*}}{n} K^{e}Sh\alpha(\Delta_{0} + H^{*}\Phi_{0} + U^{*}) - H(\omega)_{n} K^{e}Sh\alpha \left( \cos kL/2 - \frac{\sin kL/2 \sin k\Delta x}{Sh\alpha} \right) u_{0} \]  

(57)

\[ P_{n}^{*} = \frac{e^{*}}{n} K^{e}Sh\alpha(\Delta_{0} + H^{*}\Phi_{0} + U^{*}) + H(\omega)_{n} K^{e}Sh\alpha \left( \cos kL/2 - \frac{\sin kL/2 \sin k\Delta x}{Sh\alpha} \right) u_{0} \]  

(58)

It should be noted here that compressive soil pressures are supposed to be positive. The first terms of the right hand side of Eqs (57) and (58) corresponds to earth pressures induced on the sides of the foundation by the horizontal motion of the foundation when the lateral soils are free from the ground motions. These terms will result in causing out-of-phase earth pressures on the opposite sides of the foundation. The second and the third terms of these equations correspond to the earth pressures induced by ground shaking when solely the lateral soils are subjected to earthquake motions while the foundation is kept immovable. By further inspection of these terms, we will notice that the second terms of Eqs (57) and (58) will cause out-of-phase earth pressures on the opposite sides, and on the other hand the third term the in-phase pressures. This fact indicates that the third terms of Eqs (57) and (58) will cancel each other, and will not be effective as input motions into foundation from the lateral sides of the foundation.

For the case \( c \rightarrow \infty \) or \( k \rightarrow 0 \), which corresponds to the vertical incidence of seismic waves, the third terms of Eqs (57) and (58) will disappear and the phenomenon of in-phase earth pressures will never occur. The earth pressures for the group C, which contains
predominantly higher frequencies in the ground motions, have shown a close correlation with the acceleration motion of the foundation response. This tendency may be explained by inspection of Eqs (57) and (58) as follows. If considering the case $\omega \to \infty$, we will obtain as

$$K^{e}Sh\alpha \to M^{e}\omega^{2}, \quad H(\omega), K^{e}Sh\alpha \to \text{const}. $$

As a result, with increase of $\omega$ the first term will be dominant comparing to the other terms. This implies that the first terms of the equations tend to have significant effect on the earth pressures for large value of $\omega$, and to cause the out-of-phase earth pressures on the opposite sides of the foundation. It is obvious that the first terms of Eqs (57) and (58) are proportional to the acceleration response of the foundation.

Finally we will extend the discussion to the case of static earth pressures by considering $\omega \to 0$ in Eq. (44). For $\omega \to 0$, we will obtain $H_{R}(\omega) = H(\omega) \to 1$, $K^{e}Sh\alpha \to \text{const.}$ and $k \to 0$. With consideration of these limits, the expressions of the earth pressures on both sides of the foundations will be given as follows.

$$P_{R}^{e} \propto (\Delta_{0} + H^{e}\Phi_{0} + U^{*} - u_{0}), \quad P_{L}^{e} \propto -(\Delta_{0} + H^{e}\Phi_{0} + U^{*} - u_{0})$$

We will notice that $(\Delta_{0} + H^{e}\Phi_{0} + U^{*} - u_{0})$ represents the longitudinal strain at the interface between the foundation and the surrounding soil. On the other hand, the longitudinal particle velocity of soil at the interface is equal to the velocity of the foundation. Thus, reminding the fact that for one dimensional wave propagation of an elastic medium the longitudinal strain is proportional to the particle velocity of the medium (Minowa et al. 2001), the above equations may be rewritten as,

$$P_{R}^{e} \propto (\Delta_{0} + H^{e}\Phi_{0} + \dot{U}^{*}), \quad P_{L}^{e} \propto -(\Delta_{0} + H^{e}\Phi_{0} + \dot{U}^{*})$$

These expressions indicate that for lower frequencies the earth pressures will be induced in response to the velocity motion of the foundation. This may be paraphrase as the earth pressure will be induced in proportional to the relative horizontal displacement motions between the foundation and the surrounding soil.

It may be summarized that the simplified analysis model presented in this paper can successfully explain the observations of earth pressures during earthquakes.

**COMPARISON OF ANALYTICAL RESULTS WITH OBSERVATIONS**

- FORCED VIBRATION TEST-

Parameters of lumped mass system

The parameters of the multi-lumped-mass system shown in figures 9 and 13 were determined basically on the basis of the soil constants shown in figure 1. As the damping constants of soil, however, were not measured we were obliged to assume the damping factors of a hysteretic type and we assumed for the first and the second soil layers as $h_{1} = h_{2} = h$ and $h = 0.1$. Additionally, the computed results of the first to the third frequencies of the layered soil evaluated from Eq. (14) were $f_{1} = 4.4Hz, \quad f_{2} = 9.8Hz, \quad f_{3} = 18.4Hz$. Thus obtained fundamental frequency $f_{1} = 4.4Hz$ is larger than $f_{1} \approx 3.5Hz$ that was expected by observations. The discrepancy between these two may be attributed mainly
to the assumption of a rigid base underlying the surface layers. With reference to the observations, we assumed the fundamental frequency of the layered soil as $f_1 = 3.5\text{Hz}$. The other parameters of the lumped mass system are shown in Table 2. The spacing between masses was assumed to be constant all through the system as $\Delta x = 1m$.

### Table 2 Parameters of the lumped mass system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participation factor of the first mode $\beta_1$</td>
<td>$= 1.41$</td>
</tr>
<tr>
<td>Equivalent mass height $H^e$</td>
<td>$= 5.7 \text{[m]}$</td>
</tr>
<tr>
<td>Equivalent mass $M^e$</td>
<td>$= 0.92 \text{[tf \cdot sec}^2/\text{m}^2]$</td>
</tr>
<tr>
<td>Equivalent shear spring constant $K^e$</td>
<td>$= (745.5 + 149.1i) \text{[tf} \text{/m}^2]$</td>
</tr>
<tr>
<td>Equivalent axial spring constant $nK^e$</td>
<td>$= (10.9 + 2.18i) \times 10^5 \text{[tf \cdot m}^2]$</td>
</tr>
</tbody>
</table>

**Simulation of earth pressure during forced vibration tests**

Figure 15 shows the comparison of the amplitude and phase characteristics of earth pressures evaluated numerically on the basis of Eq. (33) with those of observations induced on the lateral side of foundation during the forced vibration tests. The compared results shown in figure 15 indicates that whereas the result of amplitude at WC-7 is slightly larger than the observations, the numerically evaluated results of the amplitude and phase characteristics are well corresponding to the observations as a whole.

Figure 15. Comparison of earth pressures between analyses and observations (Amplitude and phase normalized by unit displacement of foundation).

The results shown in figure 15 are the earth pressures normalized by a unit displacement of the foundation. Similarly, figure 16 shows the earth pressures normalized by a unit velocity response of the foundation. It should be reminded that the earth pressures on the lateral sides of the foundation are induced in accordance with the velocity response of the foundation for
higher frequencies more than $3 \sim 4 \text{Hz}$. This tendency is successfully explained by a simplified multi-lumped-mass model developed in this study.

Figure 16. Comparison of earth pressures between analyses and observations (Amplitude and phase characteristics normalized by unit velocity of foundation).

**NUMERICAL ANALYSIS OF EARTH PRESSURE DURING EARTHQUAKES**

**Evaluation of sway and rocking stiffness**

In order to evaluate numerically the earthquake response of the foundation on the basis of Eq. (55), it is required to obtain the sway and rocking stiffness for the foundation motions. In the numerical analyses, these values were determined on the basis of the sway and rocking responses of the foundation observed during the forced vibration tests. Let the horizontal and rocking responses of the foundation be denoted by $\Delta_0 e^{i\omega t}, \Phi_0 e^{i\omega t}$ for harmonic excitations applied on the foundation $P e^{i\omega t}$ as shown in figure 17(a). The sway and rocking stiffness of the soil underlying the foundation, $K_H$ and $K_R$, which are shown in figure 17(b), may be evaluated by

$$K_H = \frac{1}{\Delta_0} \left[ P + M_0 \omega^2 (\Delta_0 + h_g \Phi_0) - 2(\Delta_0 + H^c \Phi_0)K_L \right]$$

(59)

$$K_R = \frac{1}{\Phi_0} \left[ Ph_p + M_0 \omega^2 h_g \Delta_0 + J \omega^2 \Phi_0 - 2(\Delta_0 + H^c \Phi_0)H^c K_L \right]$$

(60)

where $h_p$ is the height of the applied force ($h_p = 7.2 \text{m}$), and $K_L$ is the lateral soil stiffness for the foundation, which is defined in Eq. (56). It should be noted that $\Delta_0, \Phi_0$ in Eqs (59) and (60) are complex valued amplitudes of the foundation. Figures 18(a) and (b) show the results of $K_H$ and $K_R$ evaluated on the basis of Eq. (59) and (60). Inspection of figure 18 reveals
that these values vary strongly with frequencies higher than $7 \sim 10\,Hz$, that is probably due to the effect of deformation of the foundation base.

**Ground motions at the foundation base**

As mentioned in the earlier section, the phase characteristics of earth pressures being induced in-phase on both sides of the foundation cannot be explained by an assumption of the vertical incidence of seismic waves. The same phenomenon was observed in observations using a small model foundation (Uchiyama et al. 1999). Uchiyama et al. (1999) tried to explain the phenomenon by using FEM and assuming a non-vertical incidence of seismic waves. Following Uchiyama et al. (1999) we assume here oblique incidence of waves. Consequently, in evaluation of the foundation responses and earth pressures during earthquakes, it is required to determine the apparent wave velocity $c$ traveling horizontally in the soil at the level of the foundation base, as well as the time history of the ground motions at that depth.

As for the apparent wave velocity $c$, the value would depend mainly on shear wave velocity of the supporting soil and on the azimuth and incident angle of seismic waves, among which the latter two cannot be decided from the observed data. A parametric study, therefore, was conducted with respect to $c$ by changing the value in the range $c = 3,000 \sim 100,000\,m/sec$. By inspection of the results of the parametric studies, $c = 10,000\,m/sec$ was chosen as the appropriate value regardless of the earthquake motions.

![Diagram of Foundation Response to Excitation](image1)

(a) Foundation Response to Excitation. (b) Resistant Model of Foundation to Excitation.

**Figure 17. Forced excitation test and equivalent resistance model of foundation.**

![Graphs of Complex Stiffness](image2)

(a) Sway Spring. (b) Rocking Spring.

**Figure 18. Complex stiffness at the base of foundation.**
On the other hand, as for the time histories of the ground motions at the level of the foundation base, which is supported at –8.2m from the soil surface, the ground motions at the depth have not been observed directly. Therefore, the ground motions at the level were evaluated numerically by means of the Thomson-Haskel’s method and with use of the ground motions recorded in the free field at the depths of –1m and –40m. The details of the evaluation of the ground motions at this site may be found elsewhere (Iguchi et al. 2000) and will not be repeated here.

Comparison of velocity response of foundation

The velocity responses of the foundation evaluated numerically on the basis of Eq. (55) are compared with observations. Figures 19(a), (b) and (c) show the compared results in terms of Fourier amplitudes between the two. As seen from the figures, fairly good agreement between two are obtained for groups A and B up to 10 Hz, whereas slight discrepancies may be recognized for group C. It was confirmed that the simplified analysis model proposed in this paper gives satisfactory results and may be applicable to numerical simulation analyses of the embedded foundation directly supported on a firm soil.

Comparison of time histories of earth pressures

Figures 20(a), (b) and (c) show the compared results of numerically evaluated time histories of the earth pressures induced on both sides of the foundation with those of observations for groups A, B and C. In the calculations, the apparent wave velocity c was assumed to be 10,000m/s. As seen from figure 20, whereas somewhat discrepancy was recognized for group C, fairly good agreement between the numerically evaluated results and observations were detected for groups A and B. One of the main reasons of the discrepancy might be attributed to uncertainties inherent to ground motions with high frequencies. It was confirmed that the multi-lumped-mass model presented in this paper is applicable not only for response analyses of a foundation but for the numerical simulation of the earth pressures induced by earthquake ground motions.

Figure 19. Comparison of velocity response of foundation: Analyses vs. observations.
Figure 20. Comparison of earth pressures on the left and right sides of foundation induced by earthquake ground motions: Analyses vs. observations.

CONCLUSIONS

The characteristics of earth pressures induced on the lateral sides of an embedded foundation during the forced vibration tests and earthquakes have been studied on the basis of
observations. In addition, a simplified analysis model, which is composed of a lumped-mass system connected in series, was presented to conduct simulation analyses of the earth pressures. The studies have led us following conclusions:

(1) The earth pressures during the forced vibration tests tend to be induced associated with the displacement motion of the foundation for frequencies lower than the fundamental frequency of the lateral soil. For frequencies higher than the fundamental frequencies of the soil, on the other hand, the earth pressures tend to be caused in-phase to the velocity motion of the foundation.

(2) The characteristics of earth pressures induced by earthquake motions are strongly dependent on frequency component included in the ground motions.

(3) The earth pressures during earthquakes are strongly related to the horizontal velocities of the foundation for rather lower frequencies and to acceleration response for higher frequencies.

(4) A phenomenon of earth pressures being induced in-phase on opposite sides of the foundation was observed for lower frequencies included in the ground motions. For higher frequencies, on the other hand, the earth pressures tend to be induced out-of-phase on the opposite sides of the foundation.

(5) A simplified analysis model presented in this paper has been proved to be effective in explanation of the observations of earth pressures induced by forced vibration tests.

(6) The analysis model could simulate satisfactorily the observations of earth pressures induced by earthquake motions except for ground motions including higher frequencies.

(7) The phenomenon of earth pressures being induced in-phase during earthquakes could be explained with use of the analysis model and by assuming oblique incidence of seismic waves.

(8) The tendencies of earth pressures on the sides of an embedded foundation being induced in close correlation with acceleration or displacement motions of a foundation could be explained theoretically with use of the simplified analysis model presented in this paper.

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REFERENCES


Forced Vibration Tests of the Foundation Block and Surrounding Soil at the NEES/UCSD Large High-Performance Shake Table

J.E. Luco, J.P. Conte, B. Moaveni, L. Mendoza, and D. Whang

ABSTRACT

A Large High-Performance Outdoor Shake Table (LHPOST) is being built at the Camp Elliot Field Station of the University of California, San Diego (UCSD) as part of the George E. Brown, Jr. Network for Earthquake Engineering Simulation (NEES). The LHP Outdoor Shake Table will provide the capability to conduct real time tests of full or large-scale structural systems including large-scale soil-foundation-structure interaction models. The table will be able to simulate near-source earthquake ground motions with large acceleration, velocity and displacement pulses. The NEES shake table in combination with a large laminar shear box and two refillable soil pits funded by the California Department of Transportation (Caltrans) will constitute a unique seismic testing facility at Camp Elliot.

The moving platen of the NEES Shake Table is 7.62 m wide, 12.19 m long, and has a weight of 130 Ton (1.275 MN). In the initial phase of the facility, the motion of the table will be uni-directional with a maximum stroke of 0.75 m, a peak horizontal velocity of 1.8 m/sec, a (bare table) peak horizontal acceleration of 4.8 g, a horizontal force capacity of 6.8 MN, an overturning moment capacity of 50 MN-m, and a vertical payload capacity of 20 MN. The testing frequency range of the table will be 0-20 Hz. The facility has been designed to be upgradeable to 6-DOF. In the initial phase the system will have two servo-controlled dynamic actuators with large servo-valves, a large power supply system (35 MPa, 9,500 liters) with 1,440 liters/min direct pumping

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capacity, an advanced real-time multi-variable controller, an innovative vertical load/overturning moment bearing system including six pressure balance bearings and two hold down struts, and a steel platen.

The reaction block/foundation for the shake table is 19.61 m wide, 33.12 m long, and extends to a depth of 5.79 m. A smaller central area of the foundation housing the hold down struts extends to a depth of 7.92 m. To reduce the mass of concrete, the corners of the block have been truncated and the structure has been designed as a hollow tube along the perimeter. A 6 m long tunnel with a 2.44 m x 2.44 m section connects the reaction block to the adjacent pump building, which is a 2-storey structure with a partial basement. The pump building has plan dimensions of 15.5 m x 22.5 m and is founded at a depth of 3.5 m. The soil pit to the east of the shake table has dimensions of 14.6 m x 15.2 m and a maximum depth of about 3 m.

The construction of the LHP Shake Table at Camp Elliot and the availability of shakers, ground motion sensors and data acquisition equipment as part of the NEES/UCLA Field Testing Facility created a unique opportunity to conduct extensive forced vibration tests of the large excavations for the reaction block, soil pit and pump building. These tests have included measurements of the dynamic response at dense arrays of locations on the excavations for the foundations, and on the soil surface up to distances of 230 m from the excavations. These tests were repeated after the reaction block and the base of the pump building had been built.

The initial set of 28 forced vibration tests of the soils within the excavation for the foundation of the NEES/UCSD Shake Table and its vicinity were conducted in April of 2003. Two NEES/UCLA shakers with a maximum force capacity of 50 Tons each were placed at opposite ends of the 45 m x 24 m x 5.8 m excavation for the reaction block of the shake table. A third, smaller UCSD shaker with a force capacity of 2.5 Tons was placed on the 18 m x 12 m x 3.5 m excavation for the adjacent pump building. Ninety-two acceleration channels and twenty-seven velocity channels were used to record the three-dimensional motion at four stations in each of the three foundation blocks for the shakers, at 27 stations in the main excavation, 7 points in the excavation for the pump building, and at 34 stations on four legs extending up to a distance of 230 m from the excavations. The tests with the main shakers were conducted at three force levels and at frequencies ranging from 0-10 Hz, 0-18 Hz, and 0-23 Hz.
for large, medium, and small forces, respectively. Ambient vibrations of the site were also recorded. The frequency range selected for the tests reflected the frequency range of operation of the table (0-20 Hz), the frequency limitations of the shakers (25 Hz), and the maximum force capacity of the shakers and corresponding foundations.

A second set of tests was conducted in October 2003 after the reaction mass for the shake table and the foundation for the adjacent auxiliary building had been completed. The tests included the use of the two NEES/UCLA MK-15 shakers mounted on the reaction mass at locations close to the reaction points of the actuators. The three-dimensional dynamic response at 19 locations on the reaction block; at 12 points on the foundation of the adjacent auxiliary building; and at 32 locations on the ground surrounding the shake table up to distances of over 230 m were recorded for longitudinal, transverse, and torsional excitation of the block.

The first objective of the experimental study was to take advantage of a limited time window to obtain dynamic ground motion data, and by inference geotechnical data that will prove invaluable in the development of a future virtual model of the complete NEES LHP Shake Table Facility. The virtual model, which will need to be exercised in preparation for any major test on the shake table or soil pit, will need to include a soil island surrounding the shake table and soil pit, and models for the reinforced concrete foundation block, the steel platen, actuators and control system, and of the test specimens. The first objective of the current research was to obtain the basic data required to develop and validate the soil island and foundation block models.

The second objective of the study was to develop a body of dynamic high-quality response data on the foundation and surrounding soil that can be used for years to come to test and validate soil-structure interaction analysis methods and computer codes. In particular, the data set will permit to test the solution to the two canonical soil-structure interaction problems: the radiation (internal source) and scattering (external source) problems. The study offers information on the coupling through the soil between adjacent foundations, a topic of considerable interest when modeling the seismic response of structures in a dense urban environment.

The third objective was to validate the unconventional design of the NEES LHP foundation block in terms of its overall dynamic response, and to study experimentally the deformability of
the foundation and surrounding soils. The conventional design of foundation reaction blocks for equipment subjected to impact loads relies on the use of massive foundations to react the impact loads essentially by inertia. In the conventional approach, the dynamic response of the foundation is reduced by designing the foundation to achieve a low characteristic frequency. In the case of the NEES foundation, the approach has been to take advantage of the natural conditions at the site in terms of high soil stiffness to design a lighter (4,300 Tons) and considerably less costly foundation which will result in a high characteristic frequency and a large effective damping ratio.

Finally, the forced vibration experiments served as a severe shakedown of the eccentric mass shakers, sensors and of the complete digitizing, recording and transmission system available through the NEES/UCLA Field Testing Facility.

The oral report presented at the workshop includes a summary of the most salient features of the recorded data and a discussion of the issues that arose during the course of the experiments. The main findings include: (i) preliminary analyses of the data obtained confirm the basic premises of the design of the facility in terms of the dynamic response of the reaction block; (ii) the data recorded during the initial tests in which the shakers were mounted on 3.7 m x 3.7 m x 1 m concrete blocks embedded in the soil indicate significant but very localized soil nonlinearities in the vicinity of the active block, while the response of the passive blocks was linear and scaled with the level of force; (iii) the results of the second phase tests show a very strong attenuation of the forced ground motion with distance from the reaction block; (iv) the data also showed a marked low frequency drift of the EpiSensor accelerometers; and (v) the excitation of unintended supra-harmonics by the shakers.
Nonlinear Soil–Structure Interaction: Foundation Uplifting and Soil Yielding

George Gazetas(1) and Marios Apostolou(2)

The study investigates the response of shallow foundations subjected to strong earthquake shaking. Nonlinear soil–foundation effects associated with large deformations due to base uplifting and soil failure are examined in comparison with the conventional linear approach. Soil behavior is represented with the elastoplastic Mohr-Coulomb model. The interplay between foundation uplifting and soil failure of the bearing capacity type is elucidated under static and dynamic conditions.

Keywords: shallow footing, soil–foundation interaction, uplift, bearing capacity, soil failure

1. INTRODUCTION

Research on seismic soil–foundation interaction in the last three decades has mostly relied on the assumption of linear (or at most equivalent-linear) viscoelastic soil behavior and fully–bonded contact between foundation and soil. Seismic design of structure foundation systems has followed a somewhat parallel path: the still prevailing “capacity design” philosophy allows substantial plastic deformation in the superstructure but requires that no significant “plastification” should take place below the ground level. This means that:

- foundation elements must remain structurally elastic (or nearly elastic)
- bearing–capacity soil failure mechanisms must not be mobilized
- sliding at the soil–foundation interface must not take place, while the amount of uplifting must be restricted to about ½ of the total contact area.

However, seismic accelerograms recorded in the last twenty years, especially during the Northridge 1994 and Kobe 1995 earthquakes, have shown that very substantial ground and spectral acceleration levels can be experienced in the near–fault zones. Seismic loads transmitted onto shallow foundations in such cases will most probably induce significant nonlinear inelastic action in the soil and soil–foundation interface. Figure 1 illustrates the three possible types of foundation–soil nonlinearity.

Observations in past earthquakes confirm the above argument. The most dramatic examples of bearing–capacity and uplifting failures of building foundations took place in the city of Adapazari, during the Kocaeli 1999 earthquake. But such phenomena are not limited to buildings: as an example of a modern monumental bridge, we mention the Rion–Antirrion cable–stayed bridge, the surface foundations of which, despite their colossal 90 m diameter, had to be designed allowing for sliding, uplifting and partial mobilization of soil rupture mechanisms to resist the prescribed high levels of seismic excitation (Pecker & Teyssandier 1998, Gazetas 2001).

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Finally, for seismically retrofitting structures that had been designed with the small acceleration levels of the past, the necessity to explicitly consider the occurrence of one or more of the above nonlinearities is often unavoidable.

The present study investigates the interplay between the two (geometric and material) nonlinearities:

- separation of the rotationally oscillating footing from the supporting soil ("uplifting"), and
- mobilization of bearing-capacity type failure surface mechanisms under large cyclic overturning moments ("soil failure").

One of the aims is to show that under seismic excitation even when the minimum (with respect to time) factor of safety (against "uplifting" or "soil failure") is well below unity, structure and foundation response may be quite satisfactory. We begin by separating the occurrence of "uplifting" from "soil failure" and even from mere "soil yielding": we study in the time domain the rocking response of a rigid block in tensionless contact with an elastic halfspace (homogeneous or layered). Although this idealized case would be directly applicable only to footings on very stiff ("non-yielding") soil, it serves as a basis for understanding the dynamics of the inelastic SSI system.
Until recently, the only preferred model to account for soil reactions on a rigid strip footing was the Winkler elastoplastic model, according to which the foundation is assumed to rotate about its center even when large angles of rotation are imposed (Allotey & Naggar, 2003, Bartlett, 1979). With the Winkler model there are two possible modes of response (illustrated in Fig 2) depending on whether the safety of factor for central vertical loading is greater or less than 2. For low values of vertical loading (FS < 2) uplift occurs at $M = N B / 6$ (point 2) before any soil yielding initiates. In this case the moment–rotation response follows the path (1)-(2)-(3)-(6). When FS > 2, soil yielding occurs when the entire base area is still in contact with soil, and the moment loading path is (1)-(4)-(5)-(6).

2. UPLIFTING AND OVERTURNING ON A VISCO-ELASTIC SOIL

Consider a foundation supported on a visco-elastic homogeneous half-space, with soil Young’s modulus $E_s$ and damping ratio $\xi$. Compared to the rocking response of a structure on perfectly rigid base, the compliance of supporting soil introduces additional degrees of freedom. The structure can now sustain rotational motion (without uplifting) for amplitudes of rotation below the critical value. The (geometrically) nonlinear nature of the problem is evident even under the assumption of an elastic soil.

Several analytical studies have already been published investigating the effect of soil compliance on rocking response of structures with foundation uplift. In these early studies (Psycharis, 1983, Chopra & Yim, 1985, Koh et al, 1986) the underlying soil was represented by distributed tensionless spring-dashpot elements. Recently, Crèmer & Pecker (2002) also analysed a foundation on inelastic continuum and developed a constitutive law to represent the uplift mechanics in an elastic or elasto-plastic soil through a single macro-element. In the present study the dynamic analysis of the rocking response is implemented with a finite element discretization using Abaqus (Hibbitt et al, 2001). The structure and the underlying soil are represented with plane-strain elements. An advanced contact algorithm has been adapted to incorporate potential slipping or uplifting of the foundation. For practical purposes the supporting soil is modelled as a homogeneous halfspace using 2D infinite elements.

We consider first a rigid rectangular structure with base width $B = 2$ m and height $H = 10$ m (aspect ratio $H / B = 5$) subjected to a base acceleration of $a = 0.30$ g. Under static conditions the moment capacity of the foundation before it overturns is:

$$M_{\text{ult}} = N B / 2$$  \hspace{1cm} (1)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{The two rocking problems studied in the paper}
\end{figure}
where \( N \) is the permanent vertical load. (The load eccentricity \( e \) corresponding to this moment is of course equal to the foundation half-width, B/2.) The maximum induced overturning moment arising from the inertial force is:

\[
M_{\text{max}} = N \left( \frac{a}{g} \right) \frac{H}{2}
\]

where \( a \) is the peak ground acceleration. For the above-mentioned structure \( M_{\text{ult}} \approx 490 \text{ kN} \) and \( M_{\text{max}} = 735 \text{ kN} \), and therefore

\[
M_{\text{max}} = 1.5 M_{\text{ult}}
\]

(3)

In static terms, such an exceedance of the (ultimate) moment capacity of the foundation would have led to toppling of the structure (factor of safety \( 1/1.5 = 0.60 \)). This is not the case however under dynamic loading: the foundation can sustain rocking motion safely even for values of the moment much higher than \( M_{\text{ult}} \). The reason: the short duration (usually a small fraction of a second) that the exceedance of the moment capacity lasts. After the uplift has started and the body is on the way to toppling, a reversal of ground acceleration makes the block decelerate, stop, and start rocking in the opposite direction. Since the natural period of a rocking block at incipient failure can be quite large, such a reversal in rocking is very likely to occur, especially with high-frequency excitation. In other words, the more “dynamic” the ground shaking the easier for a rocking structure to survive!

This paradoxical phenomenon is illuminated for the aforementioned block subjected to two different ground motions: (a) the accelerogram of Düzce (EW component, \( a = 0.37 \text{ g} \)) recorded in the Izmit 1999 Earthquake, and (b) an idealized approximation in the form of the “Ricker wavelet”, with \( a = 0.30 \text{ g} \) and dominant period \( T_E = 1.3 \text{ sec} \).

The angular displacement of the rocking foundation is computed initially for stiff supporting soil with Young’s modulus \( E_s = 100 \text{ MPa} \) and the results are plotted in Fig 4a in terms of rotation–angle time-histories. Evidently, despite the fact that \( M_{\text{max}} = 1.50 M_{\text{ult}} \), the structure undergoes rocking motion without toppling, with a maximum angle of rotation of about 0.08 rad, which is substantially lower than the critical angle for overturning under static conditions: \( \theta_c = \arctan \left( \frac{B}{H} \right) \approx 0.2 \text{ rad} \). Furthermore, the two plots for the angle \( \theta = \theta(t) \) are nearly identical, demonstrating : (i) that the simple pulse-type motion of a Ricker wavelet approximates remarkably well the essence of the Düzce accelerogram, and (ii) that the high–frequency spikes of the Düzce accelerogram do not affect the rocking response of the structure.

![Figure 4](image)

**Figure 4.** (a) The record of Düzce \((a = 0.37 \text{ g})\) from the 17–8–99 Izmit earthquake and (b) the Ricker wavelet pulse with \( a = 0.3 \text{ g} \) and \( T_E = 1.3 \text{ sec} \).

To investigate the effect of soil compliance on rocking response, the computed maximum angle of rotation, is plotted in Fig 4b for a range of \( E_s \) values (5 MPa – 1000 MPa). For very high values of the modulus of elasticity, the amplitudes of rotation converge to the limiting case of the amplitude on rigid base \((\theta_{\text{rigid}} = 0.032 \text{ rad})\). Decreasing \( E_s \) the effect of soil deformability leads understandably to greater values of the maximum angle, which can go up
to $2^{1/2}$ times the rigid base value. For even smaller values of $E_s$, less than about 10-15 MPa, the increased softening of the soil is beneficial, leading to smaller $\theta$ values! In all these cases ($E_s > 5$ MPa), the structure oscillates in rocking without overturning, despite the pseudo-statically-predicted toppling. However, for very small values of $E_s$, less than about 2 to 5 MPa, the trend changes again and $\theta$ increases with decreasing $E_s$. Failure is now possible.

![Figure 5](image)

**Figure 5** (a) Time-histories of rotation of a rectangular block-type structure with base width $B = 2$ m and height $H = 10$ m on stiff elastic half-space, (b) Peak amplitude of the angle of rotation as a function of soil Young’s models. (The critical angle for overturning under static conditions is about 0.2 rad, which is far greater than the maximum computed angle for all values of $E_s$, despite the “instantaneous” factor of safety of only 0.60.)

A parametric study is now carried out with smaller-size structures. A quite interesting rocking behavior is revealed as shown in Fig 6. Two more blocks are taken into consideration with base width 1.4 m and 1.0 m, and height 7.0 m and 5.0 m, respectively, so that, the aspect ratio is the same, $H / B = 5$, and thereby the critical angle of rotation remains also constant. In this example, it is the dimensions of each block, described through the half-diameter $R = [(B/2)^2 + (H/2)^2]^{1/2}$, that change (from 2.5 to 5.1 m).

![Figure 6](image)

**Figure 6** The peak amplitude of rotation of three rectangular block-type structures with constant aspect ratio $H/B = 5$ (equal critical overturning angle $\theta_c = 0.2$ rad) . Excitation : Ricker 0.30 g and $T_E = 1.3$ sec.

The following trends are worthy of note in this figure:

1) the overall size of the block affects strongly its rotation; the smallest of the three blocks undergoes the largest rotation for all values of $E_s$ and it in fact overturns for $E_s \approx 15$ MPa.

2) the variation of $\theta_{max}$ with $E_s$ is not monotonic; it exhibits a peak at $E_s \approx 15$ MPa – 30 MPa depending on block size, and again tends to become very large as $E_s \rightarrow 0$. A secondary peak is also noticed at $E_s \approx 150$ MPa – 200 MPa. Nevertheless, the maximum rocking angle in case of soft soil would in most cases be not more than 1.5 to 2 times the corresponding “rigid-foundation” value.
The effect of soil stiffness on rocking and especially on the overturning potential can be further illustrated using sinusoidal pulse type excitations. In Fig 7 we consider the smallest of the three studied rectangular blocks, which has a base width of 1.0 m and a height of 5.0 m (hence, $H/B = 5$, $\theta_c \approx 0.2$ rad, and $R = 2.5$ m) resting on a visco–elastic halfspace.

Under a one-cycle sinusoidal excitation of period $T_E = 0.8$ s uplifting on rigid base initiates when the ground acceleration exceeds the critical value ($\alpha_c = 0.20$ g), but the structure overtops (after one impact in the opposite direction) only when $\alpha$ has increased up to $\alpha_{over} = 0.42$ g. However, for a soft soil, with Young’s modulus $E_s = 10$ MPa the block rocks with uplifting but it does not overturn at $\alpha = 0.42$ g. The time history of the response reveals the secret of the success: thanks to its compliance, soil deforms due to moment loading in the first cycle of motion, leading to a much larger rotation ($\theta \approx 0.07$ rad) of the block than the rotation ($\theta \approx 0.02$ rad) on a rigid base. The next cycle is fatal for the block on rigid base (see the enlarged Fig 7b). For the block on soft soil, however, this strong-excitation cycle is consumed in first “arresting” the rotation towards the other side and, then, reversing the relatively large rotation ($0.07$ rad)! Thus, it cannot make it to induce but a mere $\theta_{max} \approx 0.12$ rad, which is only 60% of the required $\theta_c$ for overturning. In fact, $\alpha$ must increase to 0.84 g (doubling the previous amplitude) for overturning to occur. Eventually, if we keep increasing $\alpha$ until it reaches and exceeds 1 g, the block overturns after the very first impact.

![Figure 7](image-url)  
Figure 7  The time histories of rotation of a slender rigid block with $B = 1$ m and $H = 5$ m (critical angle $\theta_c \approx 0.2$ rad and $R = 2.5$ m), supported on elastic soil with $E_s$ as an independent parameter. The excitation is a one-cycle sinusoidal pulse with period $T_E = 0.8$ s but with different peak acceleration for each curve. (The right figure is merely an enlargement of the first 4 seconds of motion shown on the left figure.)

To further demonstrate that the role of soil compliance in overturning of a rigid block can range from very detrimental to very beneficial, we present Figure 8, which needs no further explanation.

### 3. BEARING–CAPACITY FAILURE ENVELOPE UNDER LARGE SHEAR FORCE AND OVERTURNING MOMENT

An important intermediate step in the proposed method of soil-foundation interaction analysis is the computation for a given vertical (axial) load $N$, of the combination of the limiting values of shear force $Q$ and overturning moment $M$ that will create a failure mechanism in the soil under the foundation. The problem is 3–dimensional in nature, and recent research (Butterfield 1994, Bransby & Randolph 1998, Taiebat & Carter 2000) has shown that, for any foundation shape and supporting soil, there is a surface, in load space ($N$, $Q$, $M$) independent of loading path, containing all combinations of $N$, $Q$, and $M$ that cause failure. This surface defines a bearing–capacity failure envelope for the foundation–soil system.
Figure 8  The time-histories of rotation of a slender rigid block with $B = 1 \text{ m}$ and $H = 5 \text{ m}$ (corresponding to a critical angle $\theta_c = 0.2 \text{ rad}$ and $R = 2.5 \text{ m}$), on elastic soil with $E_s$ as the independent parameter. The excitation is a one-cycle sinusoidal pulse with period $T_E = 0.8 \text{ s}$ and constant peak acceleration, $a = 0.42 \text{ g}$

While for relatively simple soil geometries such as the homogeneous halfspace limit analysis methods have provided closed-form expressions for the bearing–capacity failure envelope, for the general case of a layered profile comprising cohesive and cohesionless soils the finite element method has proved a versatile tool (e.g., Taiebat & Carter 2000).

Figure 9  Plastic deformations at imminent failure of a 6 m wide strip foundation, and cross-section of the 3–dimensional bearing capacity failure envelope. (Min kNm/unit length.)

An example of a bearing capacity failure envelop for a strip foundation on a homogeneous soil is given in Fig 9, in which the top figure portrays the contours of plastic deformations developing under the uplifted foundation, while the bottom figure plots a cross–section of the failure envelope perpendicular to the shear–force axis (Q). Note that the maximum $M_{ult}$ occurs when the vertical load is slightly less than $\frac{1}{2}$ of the ultimate vertical capacity $N_{ult}$; the Winkler model predicts exactly $\frac{1}{2}$.

4. NONLINEAR MOMENT–ROTATION RELATIONSHIP

The response of a strip shallow footing under static and dynamic loading is computed numerically with a plane–strain, finite element modeling. The footing is modeled with solid non-deformable 2-D elements. The soil is represented with solid elements and the horizontal boundaries with infinite elements. Nonlinear soil behavior is modeled with the elastoplastic M-C behavior, described by the limit state parameters $c$ and $\phi$. The superstructure is
approached with a lumped mass at the mass center, which is connected with the footing by a rigid and massless beam, so the inertia loading can be transmitted to the structural foundation. An advanced contact algorithm with gap elements has been implemented to model the interface between soil and foundation incorporating separation–uplifting.

The system we study possesses two significant degrees of freedom, namely:

- Rotation of the base about axis y (clockwise is positive)
- Vertical displacement of the base center (upward is positive)

![Figure 10 Configuration of the soil-foundation system examined in this study](image)

**Response under Static Loading: An Example**

We consider a 6 m wide, rigid strip footing, which rests on a homogenous soil layer and is initially subjected only to vertical load $N = 1000$ kN (per unit of length). The ultimate vertical capacity, $N_{ult}$, is 4000 kN (per unit of length), i.e. the factor of safety against static bearing capacity failure is $4 - a$ lightly loaded foundation. The mass of the superstructure is assumed to be concentrated at a point 12 m over the base. The parameters affecting the soil-structure system are summarized in Table 1.

A displacement-controlled force is applied at the mass centre of the structure, resulting in a horizontal and moment loading of the footing. The applied displacement is gradually increased until toppling of the structure occurs.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>System parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Soil</strong></td>
<td><strong>Foundation</strong></td>
</tr>
<tr>
<td>Elastoplastic soil stratum over rigid bedrock</td>
<td>Rigid, strip footing</td>
</tr>
<tr>
<td>Young modulus</td>
<td>20 MPa</td>
</tr>
<tr>
<td>M-C parameters</td>
<td>$c = 50$ kPa $\varphi = 30^\circ$</td>
</tr>
<tr>
<td>Ultimate vertical load (per unit of length)</td>
<td>$4000$ kN</td>
</tr>
<tr>
<td>Layer depth</td>
<td>20 m</td>
</tr>
<tr>
<td>Layer depth</td>
<td>20 m</td>
</tr>
<tr>
<td>Base width</td>
<td>6 m</td>
</tr>
<tr>
<td>Height of mass center</td>
<td>12 m</td>
</tr>
<tr>
<td>Weight (per unit of length)</td>
<td>1000 kN</td>
</tr>
<tr>
<td>Flexibility of the superstructure</td>
<td>Negligible</td>
</tr>
</tbody>
</table>
Initially, the overturning moment increases linearly with rotation angle. However, uplifting and soil yielding initiation leads to a gradually softening rocking behavior and therefore, the moment of the footing reaches a maximum value of about 1900 kNm/unit length. This value is considerably lower than the ‘elastic’ expected moment capacity \( M = \frac{N}{2} B = 3000 \text{ kNm} \) and slightly lower than the maximum ‘elastoplastic’ moment \( (\approx 2250 \text{ kNm} – \text{see Fig 9}) \). This additional reduction of the ultimate moment arises from the geometrically-induced nonlinearity of the problem.

By further increasing the imposed displacement, the overturning moment enters the declining region due to considerably increasing \( P-\delta \) effects and eventually it reaches zero at the point of marginal overturning. The complete moment–rotation monotonic curve is plotted in Fig 11 and contrasted with the curve corresponding to an infinity stiff and strong (undeformable) soil.

![Figure 11 Moment-rotation monotonic curve](image)

We repeat the above procedure for parametrically varying values of the vertical load \( N \). A quite impressive behavior is revealed: Initially when \( N \) is increased from \( N = 0.25 N_{ult} = 1000 \text{ kN} \) to \( N \approx 0.42 N_{ult} \approx 1700 \text{ kN} \), the increased static settlement of the foundation delays the initiation of uplifting and leads to a higher value of maximum moment \( M_{ult} \approx 2250 \text{ kNm} \) – the highest that can be achieved with any value of \( N \) (Fig 12). At this level of loading, the best possible combination of uplifting and soil plastification is achieved. This approximately agrees with the conclusion of Bartlet (1979) and Allotey et al (2003). However, the descending branch of the M– \( \theta \) curve drops faster, and hence the ultimate rotation at incipient overturning is slightly reduced from 0.22 rad for \( N = 1000 \text{ kN} \) to 0.20 rad for \( N = 1700 \text{ kN} \).

As the applied vertical load is further increased beyond \( N = 1900 \text{ kN/m} \) the maximum moment starts decreasing, as a result of the increased rate of plastification. The ultimate rotation angle at incipient overturning continues to decrease.

A parabolic M–\( N \) interaction diagram for the footing (a cross-section of the failure envelope) is plotted in Fig 13. The maximum moment capacity of the footing occurs when the vertical load is at or a little less than half the bearing capacity load of the supporting soil, namely at a static factor of safety of the order of 2 to 2.5.
Figure 12 Moment–rotation curves for different values of axial load N
We focus now our attention to Case B where the central vertical load is three times as large as in Case A : $N = 3000 = 0.75N_{ult}$, i.e. a heavily loaded foundation ($F.S.$ against bearing capacity failure $\approx 1.33$). The moment capacity of the foundation is $M_{ult} \approx 2000$ kNm which approximates the Case A value. However, considerable soil yielding is now taking place even for low amplitudes of rocking prior to the onset of uplift, resulting to a softer $M-\theta$ curve (see Fig 12). Moreover, the extensive plastic deformation of the soil underlying the heavily loaded foundation, amplify the $P-\delta$ effects of the declining region and result to a more ‘steep’ $M-\theta$ curve. Eventually the moment becomes zero (that means pseudostatic overturning) for an angle of rotation $\theta = 0.16$ rad, whereas for the lightly loaded foundation the corresponding angle equals to $0.22$ rad.

In Fig 14 the moment-vertical displacement curves are also plotted for both cases. In Case A the footing undergoes some small additional settlement during lateral loading until uplifting initiates. From this point on, it tends to move upwards leading to a positive displacement of the base center. In contrast, in Case B where uplifting is limited to a small portion of the base, dynamic settlement is significant and is gradually increasing during lateral loading. Eventually, at the time the overturning moment becomes zero due to $P-\delta$ effects, the downward displacement of the base center has come up to more than three times the static value. This tendency of the heavily loaded structure to respond by moving into the vertical direction as reflected by the high values of $z_c$, overshadows the uplifting potential of the foundation, up to quite high values of rotation preserving a full (or nearly full) contact condition for the soil-foundation interface.

**Response under Seismic Loading**

A long-duration Ricker pulse ($T_E = 2.2$ s, $PGA = 0.2$ g) is applied in the bedrock and is propagated through the soil to produce a free-field “input-motion” of a dominant period $T_E = 1.8$ s and $PGA = 0.32$ g as depicted in Fig 15. This long–duration type of motion resembles pseudostatically induced loading, so that the results would be comparable to those of the monotonic loading.
Figure 14  Load-deformation curves for the two loading cases under horizontal, monotonically increasing loading. The gray line corresponds to non-deformable soil.
The load–deformation relationship for the two loading cases in terms of $M–\theta$ and $M–z_c$ curves are plotted in Fig 16. The left-hand side figures refer to Case A, i.e. axial load $N = 1000 \text{ kN}$ or $\frac{1}{4} N_{ult}$, and the right-hand side to Case B, i.e. axial load $N = 3000 \text{ kN}$ or $\frac{3}{4} N_{ult}$.

Several important conclusions can be drawn from these figures:

(a) Regarding the $M–\theta$ curves of Case A, the lightly loaded foundation (F.S. = 4): The initial loading cycle follows the monotonic static $M–\theta$ curve. Upon unloading after a small excursion in the descending branch of the monotonic curve, the path follows with small deviations the original monotonic curve. This is evidence of reversible behavior – indeed the result of nonlinearly elastic, uplifting response. However, after a substantial excursion into the descending branch, unloading departs slightly from the virgin curve, as soil inelasticity is “activated” due to the large concentration of the applied normal stress when uplifting reduces substantially the area of contact.

(b) Regarding the $M–\theta$ curves of Case B, the heavily loaded foundation (F.S. = 1.33): The departure of all branches of loading–unloading–reloading cycles from the monotonic curve is far more substantial – apparently the result of strongly inelastic soil behavior. Indeed the bearing capacity failure mechanisms are fully “activated” in this case.

(c) The moment-settlement curves ($M–z_c$) echo the above $M–\theta$ response, with the curve of Case A showing the smallest deviation from the monotonic curve, and of Case B the largest.

Time histories of the response for the (rigid) structure of Case A with the hypothetical case where the foundation is perfectly bonded to the inelastic soil and hence no uplifting occurs, are plotted in Fig 17. We notice that whereas the uplifting system experiences stronger oscillatory motion in terms of angle of rotation and vertical upward displacement, it enjoys smaller levels of acceleration. The latter is cut-off at the threshold acceleration defined by the moment capacity of the foundation:

$$M = N \left( \frac{a}{g} \right) h_{cm} \cong 0.16 \text{ g.}$$
6. CONCLUSIONS

(a) The monotonic behavior of an uplifting foundation of a relatively tall structure is affected by: (i) the $P-\Delta$ phenomenon, (ii) the flexibility of the soil, and (iii) the magnitude of the normal force compared with the vertical bearing capacity of the foundation (i.e., the static factor of safety).

(b) Under seismic loading, toppling might not occur even when the instantaneous factor of safety against overturning (with bearing capacity exceedance) is well below unity. The nature of seismic excitation (specifically its frequency composition and, especially, the presence of a sequence of long duration impulsive cycles) is the controlling factor of the response of a specific system.

(c) The initiation of uplifting and the mobilization of bearing capacity “failure” can be quite beneficial for the superstructure, under certain conditions related with the fundamental period of the structure and characteristics of ground shaking.

Figure 16   Load–deformation curves ($M-\theta, M-z_c$) for the two loading cases under earthquake loading. The outcropping excitation is a long duration Ricker pulse ($PGA = 0.2$ g, $T_E = 2.2$ s). The gray lines are the monotonic loading (plus or minus) curves.
Figure 17  Time histories of the response for the case A, in conjunction with the system where soil can undergo tensile forces

7. REFERENCES


