A PRACTICAL CALCULATION OF
PILE FOUNDATION STRESSES BY SEISMIC LOADS
WITH GROUND DISPLACEMENT

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ABSTRACT

A practical procedure is proposed to calculate statically stresses produced into pile foundations from seismically induced forces, especially due to ground displacements called as the kinematic interaction problem. At first, is presented a method with key parameters of pile-soil rigidity ratio and ground displacement distribution to obtain pile stresses easy against ground displacements imposed. And the proposal for stress combination due to kinematic and inertial forces is based on the complete quadratic combination rule. This procedure is validated by simulation analyses, both of which have satisfactory agreement with each other.

Keywords pile foundation stress, ground displacement, pile-soil rigidity ratio, stress combination

INTRODUCTION

In the Hyogo-Ken Nanbu earthquake of 1995, many building foundations suffered severe damage and various failure patterns have been found from investigations after the earthquake\(^1\). One of the main causes of pile-foundation damage is inferred to be seismic actions to piles imposed by large response ground displacements of soil deposits during this earthquake\(^2\). After this, the awareness arises that the effect of ground displacements should be considered in pile-design and that must be performed to avoid such damage. From above viewpoint this paper presents a static calculation procedure to obtain pile stresses induced by earthquake response ground displacements, which aims at a practical seismic design method of pile foundations.

STANDPOINTS

During earthquakes, as is shown in Fig.1 the lateral forces from a superstructure and ground displacements of a soil deposit act on pile
foundations, which are called as inertia and kinematic problems, respectively. The stresses imposed into piles can be obtained by dynamic analyses\(^3\), however a static calculation procedure of stresses might be suitable for practical use in seismic design to determine numbers, sizes and reinforcements of piles.

Figure 2 shows the process for statically based design considered here. It is assumed in this paper that the fundamental property of earthquake input motions is defined by some standard design response spectra on the stiff soil ground with little local site effect. Therefore the response ground displacement must be statically or pseudo-dynamically estimated from the design spectra, i.e. a response spectrum method\(^4\) is applied in a sense.

The pile-stresses \(S_I\) of bending moment and shear force due to a lateral inertial force imposed at pile-top can be obtained comparatively easier in other words practically by applying the Chang’s method\(^5\) as beam model supported by an elastic soil medium. Any method corresponding to that might be required to estimate the stresses \(S_K\) against ground displacement as the kinematic problem. This is the 1st issue. The 2nd issue is on the combination rule of the \(S_I\) and \(S_K\) to set the total stresses during an earthquake for the section design of piles, because now we consider to obtain the \(S_I\) and \(S_K\) with maximum value sense by separate ways and those stresses do not occur, in general, at the same time in progress of earthquake responses. Some methods to resolve these issues are proposed in the following.

**ESTIMATION OF PILE STRESSES DUE TO GROUND DISPLACEMENTS**

The solution is obtained under the idealized condition that in the Winkler model as shown in Fig.1, the pile rigidity \((EI)\) and soil-spring \((K_{HB})\) are
uniformly constant at depth as linear system. It is assumed that the ground displacement \( U(z) \) along a long pile with \( U_0 \) at pile top level \( (z=0) \) decays exponentially on the basis of empirical judgment and the rotation of piles at top is restricted. The key parameters to express the solution are as follows.

\[
\phi = (K_H B/4EI)^{1/4}, \quad \phi = \phi \cdot Z_{uh} \quad (1)
\]

in which \( Z_{uh} \) is the depth where the \( U(Z_{uh}) \) becomes the half of \( U_0 \).

The bending moment \( M(z) \) and shear force \( Q(z) \) can be expressed as,

\[
M(z) = \phi^2 EI \cdot (\phi, z/Z_{uh}) \cdot U_0
\]
\[
Q(z) = \phi^3 EI \cdot (\phi, z/Z_{uh}) \cdot U_0 \quad (2)
\]

The functions of \( \phi (\cdot) \) and \( \phi (\cdot) \) become the results in terms with some values of \( \phi \) shown in Fig.3. Their maxima within stresses along depth are approximated as,

\[
\phi_{max} = \phi (\phi, 0) \cdot 0.38/\phi^{0.67}
\]
\[
\phi_{max} = \phi (\phi, Z_m) \cdot 0.36/\phi^{0.85} \quad (3)
\]

These characteristics give us the informations that the actual stresses from pile top to about \( Z_{uh} \) increase when the \( EI \) or \( K_H B \) becomes large and the \( Z_{uh} \) small (i.e. the variation of \( U(z) \) is large). Of course the stresses are proportional to the \( U_0 \), which with attention is strongly related to soil modulus applied for \( K_H B \) also.

**STRESS COMBINATION PROBLEM**

The complete quadratic combination rule written below has the flexibility to
express the composite maximum, $S$ of a time series consisted of 2 time history components $^{7}$.

$$S = \sqrt{S_I^2 + 2\varepsilon \cdot S_I \cdot S_K + S_K^2}$$  \hspace{1cm} (4)

where $S_I$ and $S_K$ indicate maxima of each component which are here pile-stresses obtained from each lateral top inertial force and ground displacement. The $\bar{\varepsilon}$ is called as the CQC coefficient that is evaluated on the basis of numerical studies mentioned below.

**NUMERICAL CALCULATIONS**

**Examples of Soil Deposits and Those Response Analyses**

The $S$-wave velocity ($V_S$) structures of selected three soil deposits are shown in Fig.4 and their predominant periods, $T_{S1,ela}$ in Table-1. The constitutive law for the stress-strain relationship applied is the modified R-O model expressed by the $G/Go$ and $h_s$ in terms of cyclic soil strain, the coefficients of which for each soil layer are determined from the pre-proposed empirical relations $^6$ with parameters of mean grain size (i.e. soil classification) and effective overburden pressure. The incident wave at each base-layer is the half of the earthquake input motion shown in Fig.5 (called as BCJ-L1) which is based on the design response spectra proposed by BCJ $^7$. The resultant maximum ground displacement distributions obtained by step-by-step response computations are drawn in Fig.4 which are normalized by the displacement ($U_o$) at building basement level (FB). The maximum response strain within soil deposits become 0.5, 0.5 and 0.6% in the A, B and C-Soils, respectively.

**TABLE-1**

<table>
<thead>
<tr>
<th>Soil</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Period</td>
<td>0.49sec</td>
<td>0.74sec</td>
<td>1.09sec</td>
</tr>
<tr>
<td>$U_{GL}$ *1</td>
<td>4.1cm</td>
<td>5.5cm</td>
<td>10.0cm</td>
</tr>
<tr>
<td>$\bar{\varepsilon}$ *2</td>
<td>0.13m$^{-1}$</td>
<td>0.18m$^{-1}$</td>
<td>0.14m$^{-1}$</td>
</tr>
</tbody>
</table>

*1 Peak Displacement at GL by BCJ-L1  
*2 $\bar{\varepsilon}$ for Single Pile of 2m-Diameter
response spectra obtained from the response accelerations at each G.L. have the predominant periods caused by soil nonlinearity are 0.8, 1.2 and 1.8 seconds for each soil\textsuperscript{4}).

**Pile Stresses against Ground Displacement U(z)**

The RC-pile foundations with the diameter of 2 meters for numerical examples are settled into either soil deposit of A, B or C-soil shown in Fig. 4 and embedded into base-layer. The distributions of U(z) used are response results at depth below FB shown in Fig. 4. The $K_H B$ for each discretized portion of single-pile is determined by the form of $f_E \cdot Geq$, in which the $f_E$ is 2.5 to 3.0 at the upper of pile and the $Geq$ to transform equivalent linear $K_H B$ means reduced soil modulus corresponding to effective strain of 65% of response peak strains. In the other case of 7 \textsuperscript{7} pile group, the sum of $K_HB$ for single pile is decreased to 1/7, while for the EI the sum is kept. Taking the average of $K_HB$ from top to Zuh, the $f$ of single pile by Eq.(1) are shown in Table-1 and the $f$ become 0.5, 2.7 and 1.9 in each soil. Under these conditions, the stresses of piles pin-supported at bottom are computed using the discrete model of Fig.1. The results normalized similar to Fig.3 are shown in Fig.6, where the subscript (g) indicates group pile. The numerically computed bending moments at the portions of $M>0$ can be relatively well expressed by the results of Fig.3 and Eq.(3). The other stress manners point out the necessity using the original $K_HB$ different from the average.

**CQC Coefficients, $f$ for Stress Combination**

Case studies are carried out to quantify the $f$ in coupled building-pile responses. A building-pile model is schematically illustrated in Fig.7. Three RC-building models shown in Table-2 are used and represented by lumped
mass system with inelastic shear spring and linear flexural spring. The 1st period in Table-2 means the fundamental period in elastic range under the fixed-base condition at the 1st floor. The Takeda model of degrading tri-linear type is applied for restoring force characteristics of shear spring with 33% cracking strength to yielding. The weight of basement is 5000tonf and pile foundations are connected with the bottom of basement floor at the depth of GL-7.5m. These building-pile systems are settled into 3 kinds of soil deposit shown in Fig.4, therefore computational cases are 3\times3. The response analyses of building-pile coupled motions are conducted by so called the Penzien-type model as coupled discrete system. The pile and soil-spring are modeled into the same as ones described above. The earthquake input motion on the exposed base-layer is BCJ-L1 shown in Fig.5.

<table>
<thead>
<tr>
<th>Upper Structure</th>
<th>T05</th>
<th>T10</th>
<th>T15</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Period</td>
<td>0.5sec</td>
<td>1.0sec</td>
<td>1.5sec</td>
</tr>
<tr>
<td>Story Number</td>
<td>8-story</td>
<td>16-story</td>
<td>24-story</td>
</tr>
<tr>
<td>Strength Coef. *1</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Diameter of Pile</td>
<td>1.6m</td>
<td>2.0m</td>
<td>2.4m</td>
</tr>
<tr>
<td>Shear Coef. *2</td>
<td>0.18</td>
<td>0.15</td>
<td>0.09</td>
</tr>
</tbody>
</table>

*1 Yield Story Shear Strength Coef. of 1st Floor
*2 Peak Shear Force Coef. of 1st Floor by BCJ-L1

Fig.7

Fig.8 Examples of maximum bending moments

Fig.9 CQC coefficients, □
The pile stresses, $S$ as the total are first obtained following the response analysis models mentioned above. Next, removing the superstructure with basement floor, only pile-models are analyzed where the maximum stresses are set to the $S_K$. The stresses $S_I$ due to inertia forces are assumed to be maxima of time histories neglecting above pile only responses from the coupled model responses. Figure 8 shows the examples for bending moments of piles in the cases of T10-building supported by A or C-soils. Taking the absolute values of the maximum stresses, the CQC coefficients, are inversely calculated from the relation of Eq.(4) and the results are shown in Fig.9. In these $\tilde{\alpha}$, the values of -0.5 to -1.0 are little meaningful because these cases become $M_I \leq M_K$. The attention should be paid to upper portion of piles in the cases of T05-building in B or C-soil, the $\tilde{\alpha}$ in which become almost +1 because of superposition effect of short and long period responses. Except these comparatively special cases, it might be desirable that we take the $\tilde{\alpha}$ of 0.0 (SRSS rule) to 0.3 in the combination of Eq.(4).

CONCLUSIONS

A static calculation procedure of pile stresses is proposed for the practical seismic design. The proposal is consisted of the estimation methods of 1) pile stresses (kinematic) against ground displacements, and 2) composite stresses from inertial and kinematic problems for combination. Through the simulation analyses it can be concluded that although other problems still remain, these methods have rather good performance for the estimation.

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