

# STRUCTURAL RESPONSE CHARACTERISTICS IN CONSIDERATION OF DIRECTIONAL UNCERTAINTY OF EARTHQUAKE GROUND MOTION

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## Abstract

This paper presents structural response characteristics in consideration of the directional uncertainty of earthquake ground motion. Bilateral Acceleration Response Spectrum (BARS) for SDOF system was compared with unilateral one (UARS) to evaluate the variation of ARS caused by the directional uncertainty. The results showed that BARS was 1.2-1.3 times larger than UARS which was calculated from the unilateral component for arbitrary direction. Furthermore, the method to estimate BARS that were based on the Square Root of Sum of Squares (SRSS) combination rule and Percentage combination rule was proposed. It was found that proposed methods estimated BARS well.

## Introduction

Earthquake ground motion is consisted of three orthogonal components, i.e., two horizontal components and vertical component, the structure responds in three-dimensional behavior. However, the structure is generally designed so that the seismic loads are applied independently for the principal axes of the structure (for example, bridge axis and perpendicular bridge axis). Moreover, the impact of the input direction of ground motion cannot be known without carrying out trial and error analyses. Therefore, the structure might not satisfy the strength demand due to underestimation of the structural response even if the analytical model could simulate the structural response perfectly.

Several seismic design specifications prescribe the combination method to take this effect into consideration. ISO 3010<sup>1)</sup> prescribes that the total design seismic action  $E$  is obtained from seismic actions  $E_x$  and  $E_y$  for two orthogonal horizontal components of the structure as follow;

$$E = \sqrt{E_x^2 + 2\varepsilon E_x E_y + E_y^2} \quad (1)$$

where,  $\varepsilon$ : coefficient (from -1 to 1, empirically taken as 0 to 0.3)

If  $\varepsilon=0$  at Eq.(1), this corresponds to the Square Root of Sum of Squares (SRSS) combination rule method.

As the first order approximation of Eq.(1), Eq.(2a) and Eq.(2b) are obtained as;

$$E = E_x + \alpha E_y \quad (2a)$$

$$E = \alpha E_x + E_y \quad (2b)$$

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where,  $\alpha$ : rate of premium (taken as 0.3 to 0.5)

This combination rule, generally called “Percentage combination rule”, is specified in many specifications, though rate of premium  $\alpha$  are different. For example,  $\alpha$  is prescribed as 0.3 (30%) in AASHTO Standard Specifications<sup>2)</sup> and Caltrans Seismic Design Criteria<sup>3)</sup>, 0.4 (40%) in ATC-32<sup>4)</sup>. Eurocode 8<sup>5)</sup> is also applied for either formula ( $\alpha$  is taken as 0.3 for Eq.(2a), (2b)) including the vertical component which is usually not considered in many specifications.

*Wilson et al.*<sup>6)</sup> recommended the SRSS combination rule because it could be easily obtained by multiplying an additional factor which gave greater result than the input spectra for arbitrary direction and it gave conservative results though the percentage combination rule didn’t cause major errors.

*Menun and Kiureghian*<sup>7)</sup> proposed the extension method of the CQC combination rule (called “CQC3” in Reference 7)), which is commonly used to combine the modal responses resulting from the arbitral directional component of ground motion. They indicated that the CQC3 method should be adopted as a general rule because this method covered the current methods as one of the special cases and it can be easily calculated.

However, it is not clear which combination rule is appropriate to obtain the total response of the structure. The purpose of this study is to propose the method to estimate the maximum acceleration response of elastic structure in the horizontal plane based on the statistical analysis.

### **Definition of UARS and BARS**

To focus on the effect of directional uncertainty of ground motion, the structure is simply modeled as SDOF system that has the isotropic elastic stiffness in the horizontal plane as a column with circular cross section. Natural periods are assumed as 0.1-5.0 second with every 0.1 second and damping ratio is assumed as 5%.

The acceleration waveforms of ground motion for arbitrary direction are calculated by two orthogonal horizontal components of earthquake ground motion records as shown in Figure1. They are obtained by rotating original waveforms every 15 degrees between 0 and 180 degree for coordinate axis (so, 12 waveforms are given by each original earthquake record) and unilateral acceleration response spectrum (UARS) are calculated for these waveforms.

It is assumed that the acceleration responses for two orthogonal components be independent in the elastic response because the effect of bilateral bending on the structural response is generally small. Hence, the bilateral acceleration response spectrum (BARS) is defined as maximum magnitude of vector sum of orthogonal unilateral acceleration responses as Eq.(3);

$$BARS(T) = \max \sqrt{(UARS_x(T)|_t)^2 + (UARS_y(T)|_t)^2} \quad (3)$$

where

$T$  : Natural period of SDOF system (second)

$UARS|_t$ : Unilateral acceleration response at time  $t$

$x,y$  : Coordinate axes

Accurate maximum value of SDOF system can be obtained for this equation directly, even if the direction at BARS is different from coordinate axes of the input ground motion.

Following 5 large earthquake ground motion records are used for this analysis; JMA Kobe, JR Takatori Station<sup>8)</sup> and Higashi Kobe Bridge records during 1995 Hyogo-ken Nanbu Earthquake, JMA Kushiro record during 1993 Off Kushiro Earthquake and Sylmar record during 1994 Northridge Earthquake.

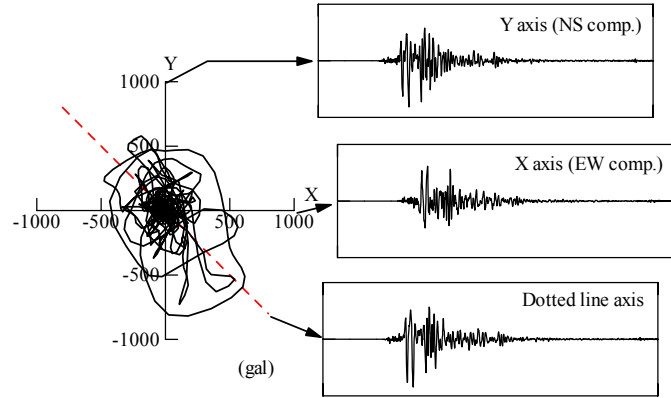


Fig. 1 Acceleration waveform for each principal axis (JMA Kobe at 1995 Hyogo-ken Nanbu earthquake)

### Methodology of BARS estimation

Only one component waveform or the seismic load is commonly used for the structural design, it is necessary to estimate the value corresponding to BARS without using two orthogonal horizontal components of waveforms.

In this study, 2 cases are assumed; UARS is given only for arbitrary directional component (Case1), and is given for two orthogonal horizontal components (Case2).

In Case1, estimated BARS (EBARS1) is assumed using UARS with coefficient  $\beta$  as,

$$EBARS1(T) = \beta \cdot UARS(T) \quad (4)$$

In Case2, 2 methods referred to current estimation methods are assumed.

a) SRSS combination method

$$SRSS(T) = \sqrt{(UARS_1(T))^2 + (UARS_2(T))^2} \quad (5)$$

where, 1,2 (subindex): axis1 and axis2 (orthogonal axes)

b) Percentage combination method (PARS)

$$PARS1(T) = UARS_1(T) + \alpha \cdot UARS_2(T) \quad (6a)$$

$$PARS2(T) = \alpha \cdot UARS_1(T) + UARS_2(T) \quad (6b)$$

Where, rate of premium  $\alpha$  is assumed as 0.3. PARS is defined as the maximum value of these equations for each natural period of SDOF system.

$$PARS(T) = \max(PARS1(T), PARS2(T)) \quad (6c)$$

Spectral ratio (SR), which is defined as Eq.(7), is used to remove bias caused by the difference of absolute value.

$$SR = EBARS(T)/BARS(T) \quad (7)$$

where, *EBARS*: estimated BARS by EBARS1(=UARS) or SRSS or PARS

Statistical analyses for the spectral ratios are carried out to evaluate the accuracy of estimated values for above 3 methods.

Assuming that the variation of EBARS and BARS depends on the normal distribution, the variation of SR depends on the lognormal distribution. The logarithmic mean  $\lambda$  and the logarithmic standard deviation  $\zeta$  are obtained at each natural period for each case. Coefficient of variation (CV) is useful to evaluate the variation of the estimation methods. Hence, CV can be derived from Eq.(9) on the basis of the relation between the normal distribution parameters and the lognormal distribution parameters as Eq.(8a) and Eq.(8b),

$$\mu = \exp(\lambda + \zeta^2/2) \quad (8a)$$

$$\sigma = \mu \sqrt{\exp(\zeta^2) - 1} \quad (8b)$$

$$CV = \sigma/\mu \quad (9)$$

where

$\mu$  : mean of the normal distribution

$\sigma$  : standard deviation of the normal distribution

Based on the results of the statistical analysis, modification coefficients for 3 methods are proposed to obtain the maximum structural acceleration response value of elastic SDOF system.

### **Principal axis analysis of earthquake ground motion**

To examine the relationship between the predominant shaking direction of ground motion and the maximum structural response direction, the eigen value analysis for ground motion by Penzien and Watabe technique<sup>9)</sup> is carried out.

Eigen value is found by Eq.(10) for the matrix defined as Eq.(11).

$$Ea = \gamma^2 a \quad (10)$$

$$E = \sum_{i=1}^n \begin{bmatrix} x_i x_i & x_i y_i \\ y_i x_i & y_i y_i \end{bmatrix} \quad (11)$$

where,

$\gamma$  : eigen value

$a$  : eigen vector

$x_i, y_i$ : acceleration at  $i$ -th data for x component and y component

$n$  : number of data for ground motion record

The eigen value and the eigen vector are obtained for 2 pairs, ( $\gamma_1, a_1$ ) and ( $\gamma_2, a_2$ ), where  $\gamma_1$  is larger than  $\gamma_2$ . Therefore, strong axis is given by  $\gamma_1 \times a_1$  and weak axis is given by  $\gamma_2 \times a_2$ .

## **Comparison between BARS and UARS**

Figure 2 shows the comparison between BARS and UARS. The maximum response direction for each natural period and the principal axis of the earthquake ground motion are also shown in these figures. The values of UARS at the same natural period are varied widely by the input direction of the earthquake ground motion, especially in the vicinity of peak natural period of BARS. For example, the maximum UARS at the natural period of 0.8 second obtained from JMA Kobe record is about 3 times larger than the minimum one.

It is often that the correlation between maximum response directions and principal axis of earthquake ground motions is not good, especially long natural period range and Sylmar record case. This result means that BARS is unable to be estimated by only one UARS for arbitrary directional component.

## **Comparison between BARS and combination methods (SRSS, PARS)**

Figure 3 shows the comparison between BARS and SRSS or PARS. SRSS and PARS, which are calculated from 6 orthogonal UARS pairs, are also shown in these figures.

Spectral shapes of SRSS or PARS are similar to those of BARS for all natural period range of SDOF system. Spectral variations of SRSS or PARS for 6 UARS combination pairs are smaller than that of UARS cases. Moreover, the absolute response values of SRSS or PARS are nearly equal to those of BARS in long natural period range. These results indicate that BARS can be estimated well using SRSS or PARS.

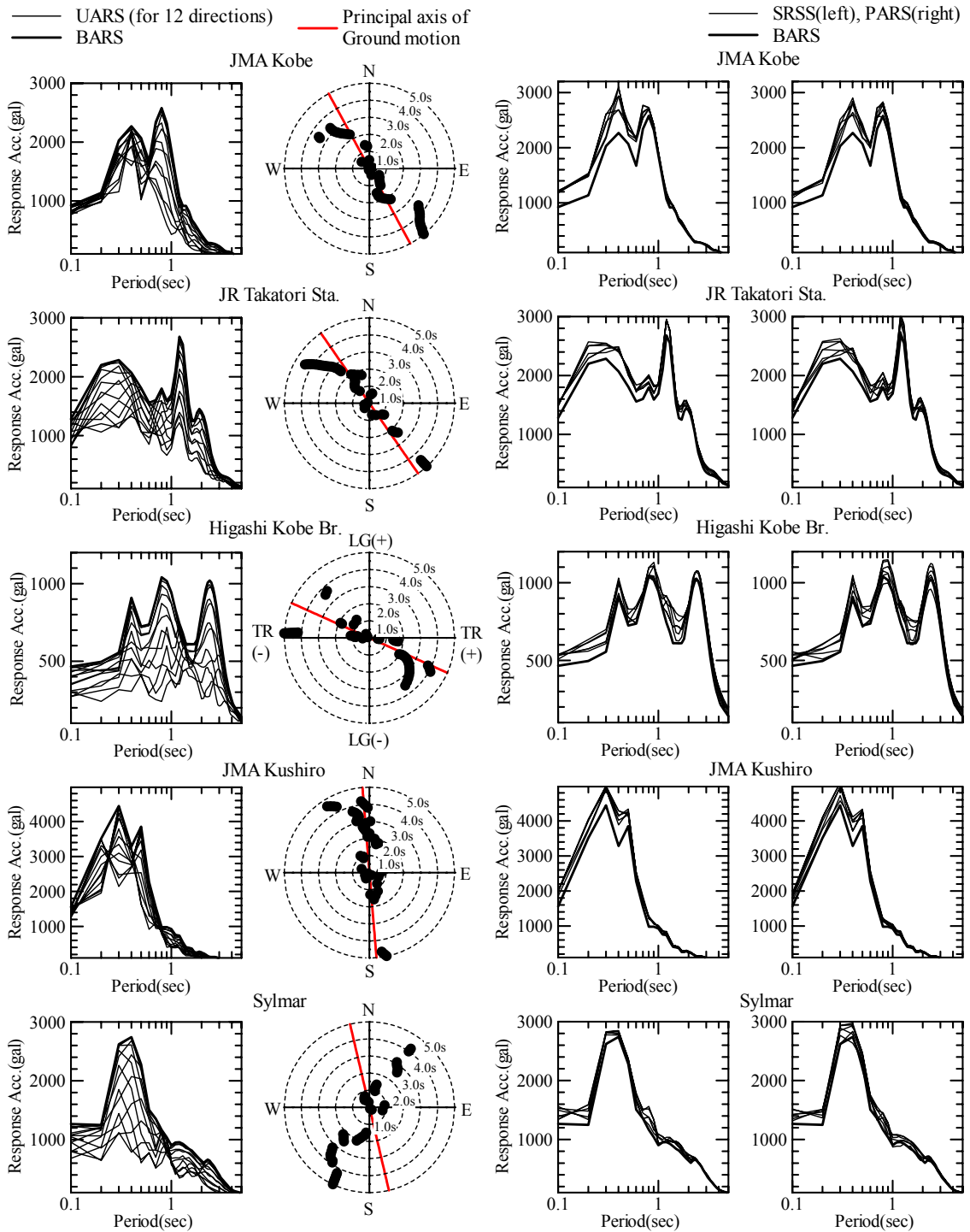
## **Statistic analysis of spectral ratios**

Figure 4 shows the spectral ratios to BARS of UARS (UARS/BARS) or SRSS (SRSS/BARS) or PARS (PARS/BARS) for each record. Figure 5 shows the logarithmic mean  $\lambda$  and the logarithmic standard deviation  $\zeta$  of spectral ratios.

Variation of UARS/BARS is large in comparison with the other types of spectral ratios. The margin of fluctuation of  $\lambda$  of UARS/BARS is from -0.4 to -0.1 and that of  $\zeta$  is from 0.1 to 0.5 for each earthquake ground motion record. In SRSS/BARS and PARS/BARS, these values are from 0.0 to 0.3 for  $\lambda$  and from 0 to 0.1 for  $\zeta$ .

The most characteristic thing is that the margin of fluctuation of  $\zeta$  of UARS/BARS is obviously larger than that of SRSS/BARS and PARS/BARS. Moreover,  $\zeta$ s of SRSS/BARS and PARS/BARS are stable for all natural period range independently of the difference of earthquake ground motion records. The variation of  $\lambda$  and  $\zeta$  caused by the difference of records are generally small except for the JMA Kobe record case. In this case,  $\lambda$  becomes small in proportion as natural period of the system becomes long.

$\lambda$  and CV calculated for all records are shown in Figure 6.  $\lambda$  of SRSS/BARS and PARS/BARS are stable and the values are about 0.1 for all natural periods. This trend is also found UARS/BARS case ( $\lambda$  is approximately -0.3). CVs of SRSS/BARS and PARS/BARS are also stable and small (approximately 5-8%). On the contrary, CV of UARS/BARS tends to become large as natural period becomes long (approximately 15-40%).



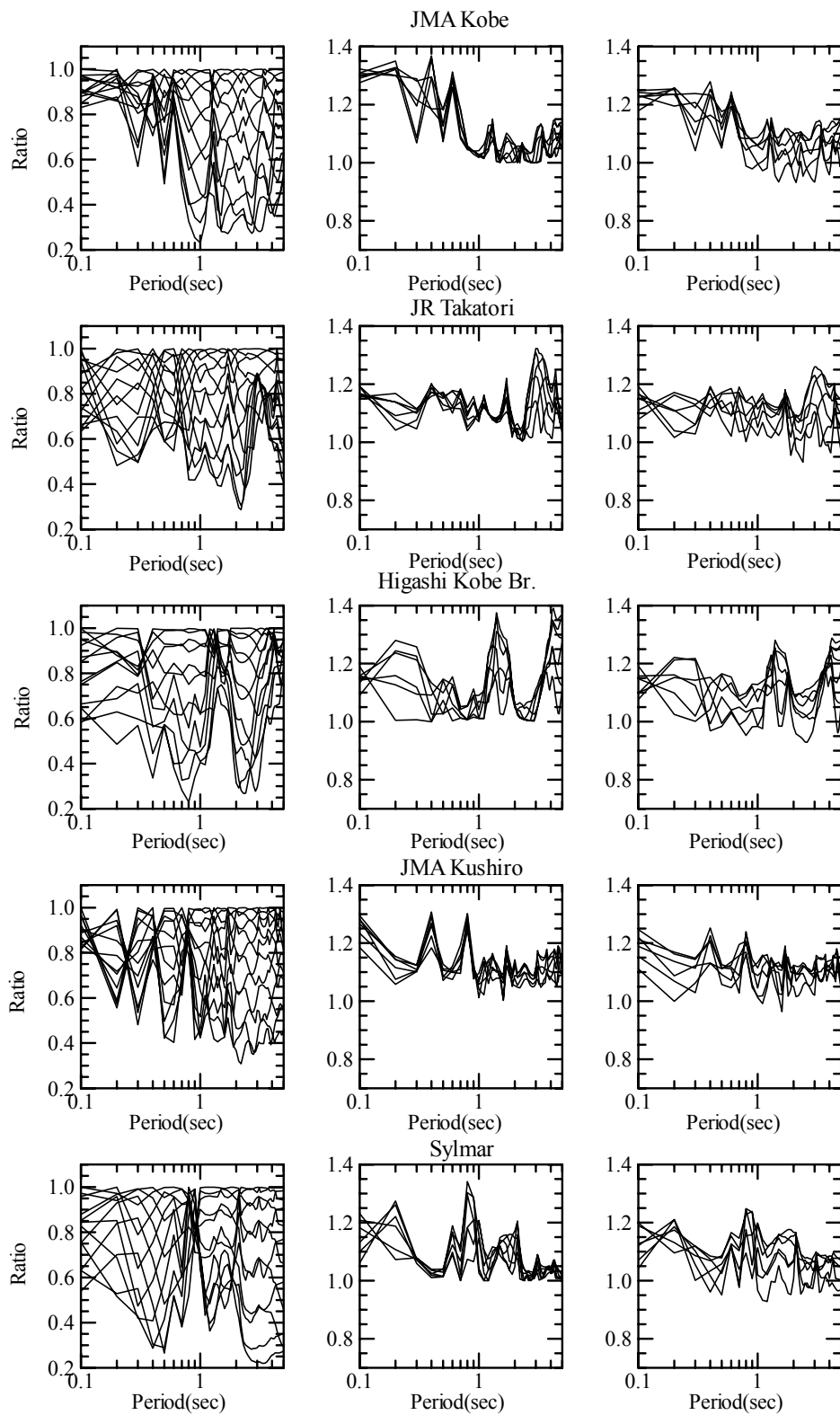


Fig. 4 Comparison of spectral ratios  
 (Left: UARS/BARS Center: SRSS/BARS Right: PARS /BARS)

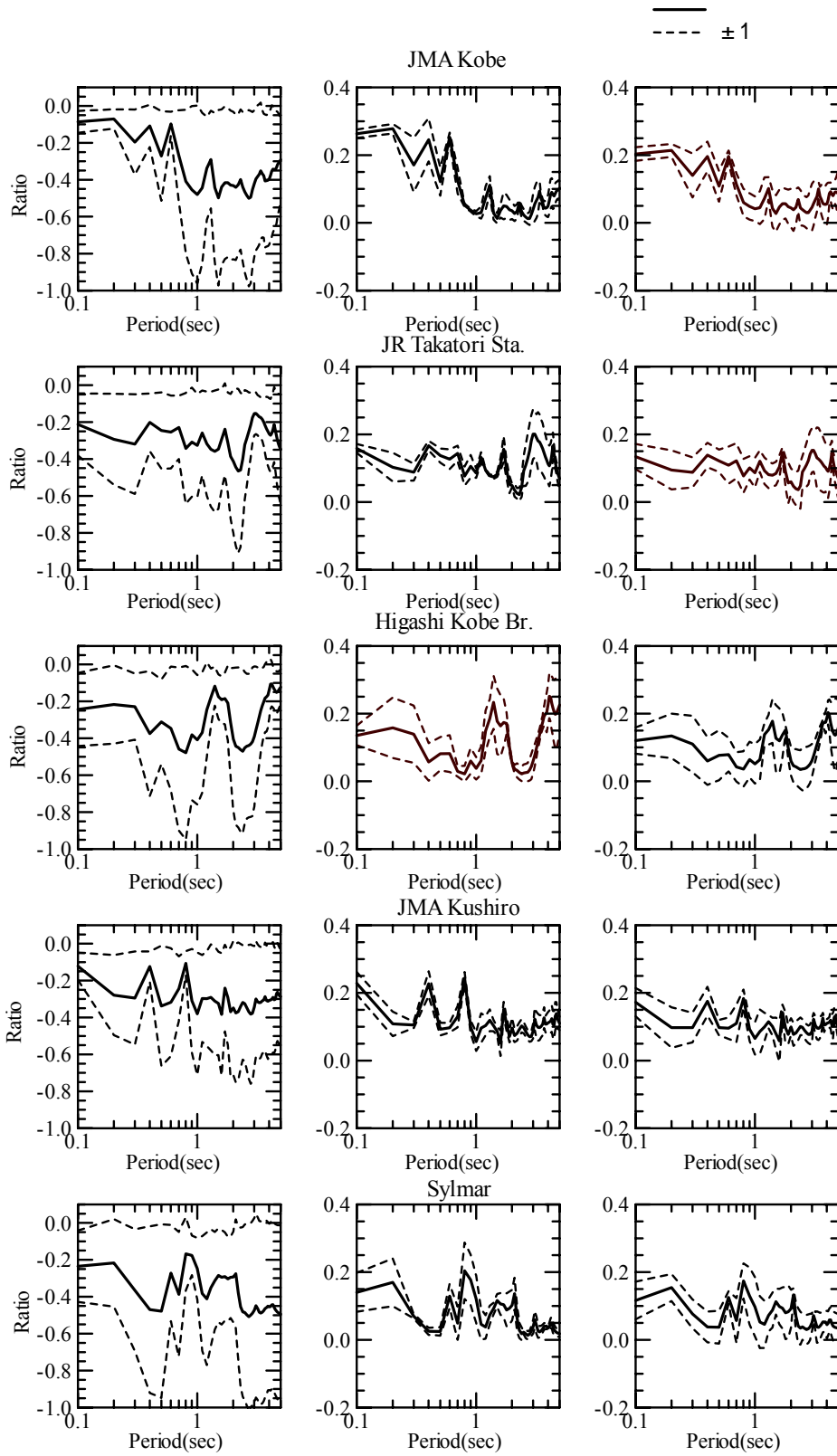


Fig. 5 Logarithmic average and standard deviation of spectral ratios  
 (Left: UARS/BARS    Center: SRSS/BARS    Right: PARS /BARS)



These results show that BARS can be estimated well based on the 3 methods though the accuracy of estimated values are different for the estimation methods.

### **Modification of estimation methods**

Since the logarithmic mean  $\lambda$  is stable for all natural periods, modification coefficient is proposed as fixed value for each method. Then, modification coefficient is given based on  $\lambda$  as follows; 0.75 for EBARS1 ( $\lambda=-0.3$ ), 1.10 for SRSS and PARS ( $\lambda=0.1$ ).

To compare EBARSs by the difference of the estimation methods simply, it is assumed that UARS for arbitrary direction is equal to that for perpendicular direction at natural period  $T$ . The estimation results are;

$$EBARS(T)_{EBARS1} = \frac{UARS(T)}{0.75} = 1.33UARS(T) \quad (EBARS1) \quad (12)$$

$$EBARS(T)_{SRSS} = \frac{\sqrt{2}UARS(T)}{1.10} = 1.29UARS(T) \quad (SRSS) \quad (13)$$

$$EBARS(T)_{PARS} = \frac{1.3UARS(T)}{1.10} = 1.18UARS(T) \quad (PARS) \quad (14)$$

EBARS for each method is 1.2-1.3 times larger than UARS. However, accuracy of estimated values are different by the estimation methods that mentioned above, the reliability of estimated EBARSs by using two orthogonal unilateral acceleration responses (SRSS, PARS) is higher than that by using one unilateral acceleration response (EBARS1).

### **Conclusions**

Due to directional uncertainty of ground motion, acceleration response for arbitrary direction is not always maximum one in the horizontal plane. To estimate maximum acceleration response in the horizontal plane, bilateral acceleration response spectrum for SDOF system (BARS) is calculated and the estimation methods are proposed based on statistical analysis.

The results of this study are concluded as;

- (1) BARS can be estimated well by proposed 3 estimation methods; EBARS1 method,

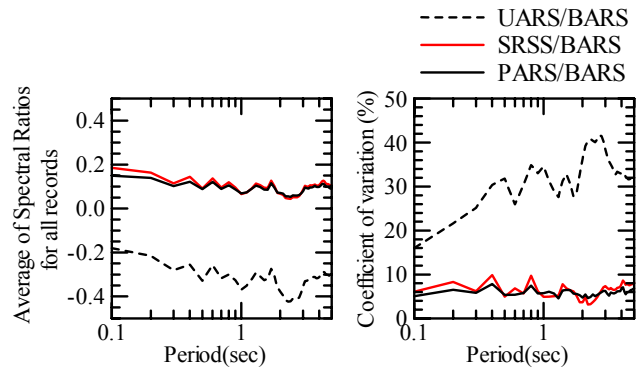


Fig. 6 Comparison of spectral ratios for 5 records (Left: Average Right :Coefficient of variation)

Square Root of Sum of Squares (SRSS) rule method and Percentage rule method (PARS).

- (2) Estimated values of BARS for each method (EBARS) are 1.2-1.3 times larger than UARS. However, accuracy of estimated values are different for the estimation methods, the reliability of estimated EBARSs by using two orthogonal unilateral acceleration responses (SRSS, PARS) is higher than that by using one unilateral acceleration response (EBARS1).

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