

ON DESIGN OF HIGHWAY BRIDGES AGAINST UNINTENTIONAL HAZARDS AND MALICIOUS ATTACKS

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Abstract

Hazard events can generally be classified in two major types: unintentional (Natural and Accidental) hazards and malicious attacks. The distinct nature of the two types of hazards implies that the hazard associated uncertainties, severity and frequency of occurrences are significantly different. Design of highway bridges for the multi-hazard faces a general challenge of commensurable risk quantification. For the unintentional hazards such as earthquake, wind, scour, vessel collision, random stochastic models are typically used to represent the hazard intensity and occurrence. A reliability based approach can be applied to determine reasonable loading combinations. An example of optimal life-cycle cost design method is explored. For purposely plotted malicious destruction such as explosion and intentional collision and purposely made accidents, the ordinary random stochastic model is no longer valid. New approach must be developed in that it typically has to consider various uncertainties such as intent, method of attack, possible consequences, and difficulty of penetration. Even then, a model will still be difficult to provide accurate risk quantification. Some possible design alternatives are discussed.

Introduction

Despite the fact that severe hazards are rare events, the distinct nature of the two types of hazards, unintentional hazards and malicious attacks, implies that the hazard associated uncertainties, severity and frequency of occurrences are significantly different. Design of highway bridges for the multi-hazard faces a general challenge of commensurable risk quantification. For the unintentional hazards, random stochastic models are typically used to represent the hazard intensity and occurrence. A reliability based approach can be applied to calibrate load resist factors for LRFD load combinations. This approach provides a commensurable assessment of the risks associated to various hazard occurrences and corresponding load combinations.

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Its application, however, is limited to statistically predictable hazards. For purposely plotted malicious destruction such as explosion and intentional collision and purposely made accidents, the ordinary random stochastic model is no longer valid. A risk evaluation model typically considers various adaptive uncertainties such as intent, method of attack, possible consequences, and difficulty of penetration. Even then, a model will still be difficult to provide satisfactory risk quantification. Therefore, some new approach must be developed.

Reliability-based Approach to Determine Unintentional Hazard Loading Combinations

Design of highway bridges must consider various unintentional hazards including earthquake, wind, flood, vessel collision and scour, for the entire life span of the bridge. In general, the bridge design against multiple unintentional hazards covers two types of hazard occurrences: single hazard and simultaneous multiple hazards. Since hazard loadings are additional loads while the bridge is under the normal operational loads (dead load plus live load) and the probability of hazard occurrence is low but statistically predictable, the potential hazard loadings are considered traditionally by using simplified rules such as one-third stress-reduction. Such treatment lacks a common ground to judge the capability of a bridge to resist different hazards, and is even less satisfactory in handling multiple hazards occurring simultaneously. The current AASHTO LRFD specification has adopted a reliability based methodology to calibrate the dead load plus live load. However, other load combinations are inherited from previous version of the specification, which may or may not be reliability based. Therefore, the target reliability levels for different hazards may not be consistent. An effort has been given to develop the current reliability based methodology for a uniform safety indexes for multiple hazard design (NCHRP 489).

General reliability theory provides a way to deal with the uncertainties associated with structural safety under statistically predictable multiple hazards. If the interested object X can be modeled as a random variable, the uncertainty is modeled as the coefficient of covariant

$$COV_x = \frac{\sigma_x}{\bar{X}}$$

where σ_x is the standard deviation, \bar{X} is the mean of X. This measurement represents uncertainty as the dispersion of the randomness of X.

In structural reliability analysis, safety can be described as the situation in which resistance capacity (strength, resistance, fatigue life, etc.) exceeds demand (load, moment, stress ranges, etc.). A limit state equation for a bridge is the margin of safety for any type of failure mode in a deterministic fashion such that it is clear from the

value of the limit state variable, Z , whether the component has survived or failed. For example, define the random variable Z as a margin of safety as follows (NCHRP 489):

$$Z = R - S$$

where R is the resistance or member capacity and S is the load effect. Probability of failure of the bridge, P_F , is the probability that the resistance R is less than or equal to the total applied load effect S or the probability that Z is less than or equal to zero. This is expressed by the following equation:

$$P_F = \text{Prob}(R \leq S)$$

If R and S follow independent normal distributions, then

$$P_f = \Phi\left(\frac{0 - \bar{Z}}{\sigma_z}\right) = \Phi\left(-\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right)$$

where Φ is the normal probability function that gives the probability that the normalized random variable is below a given value, \bar{Z} is the mean safety margin,

μ_R and μ_S are the mean of the resistance and the mean of the load respectively,

$\sigma_z, \sigma_R, \sigma_S$ are the standard deviation of the safety margin, the standard deviation of the resistance, and the standard deviation of the load, respectively.

The reliability index β is defined as $P_F = \Phi(-\beta)$. Again for normal distribution,

$$\beta = \frac{\bar{Z}}{\sigma_z} = \frac{\bar{R} - \bar{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

which is the reciprocal of coefficient of covariant COV_Z . Note that if the uncertainty can be represented by COV, then high COV value will result in low reliability index β . It follows that the for statistically unpredictable hazard events, improving reliability is practically impossible.

Although β is originally defined for normal distributions, the definition can be extended to other distributions using FORM (first order reliabilities methods) and SORM (second order reliability methods). Therefore, given a specific failure probability P_F , the reliability index β can be computed from the probability accumulative function, and it is used as the safety measurement or safety index. The commonly accepted range of reliability index in many design codes and specifications is between 2 to 4, and it is usually specified for different structural applications. For instance, AASHTO LRFD uses $\beta = 3.5$ for calibration of basic

strength limit state with 1.25DL +1.75LL, one of the seven load combinations prescribed in the AASHTO specifications.

Commeasurable Risk Assessment

The current use of reliability index β is mainly for calibration of appropriate load combinations and resistance safety factors. The recommended combination and factors are primarily for new structural design. The calibration is carried out against existing safety criteria already established in previous versions of the codes or practices. The difficulty of directly applying β in structural design is rooted in the commensurable risk assessment for multiple hazards.

The fundamental problem of commensurability in bridge design against multiple hazards is the various types of uncertainties associated with different hazards, hazard resistance of a bridge, failure modes of bridge under hazard loadings, and consequences of a hazard instance or disaster. In view of the different characters of uncertainties, a commensurable risk assessment must first establish criteria of risk comparison, which in turn have to do with the practical objectives of the risk assessment. For example, if we only consider the comparison of the occurrence of various hazards, a simple commensurable risk assessment criterion can be based on the return period or probability of exceedence in a given time period. In current AASHTO specifications, the design hazard for earthquake is set at 475 year of return period, which has a 10% probability of exceedence in 50 years. Recent recommendation from NEHRP (NEHRP 2000) intends to increase the return period to 2500 years. Wind hazard is set at 50 year of return period according to ASCE 07-95. Scour is set at 100 year return period following FHWA HEC 18. For comparison, live load is set at the maximum of 75 years, the design life-span of the bridge. The intention of hazard classification based on a commensurable return period is to set a reference for design loadings of individual hazards. In this regards, the commensurable criterion has only considered the uncertainties of the hazard occurrence; but not the cause of hazards and the resistance of the structure. Vessel collision, for example, is another hazard which cannot be properly modeled by return period since the vessel size and weight in a particular river is physically restricted; therefore, the impact from collision will not significantly vary from one year to the next. It is measured as the rate of total collision incidents per year regardless of the location of the incidences.

Reliability can be another commensurable criterion that considers the hazard loading demand and the structural resistance capacity. The judgment involves more contributing factors and is subject to more uncertainties. One of the challenges is to model the combined effect of simultaneous hazards because the most critical situation of structural reliability is when several hazards occurred at the same moment. Three

different methods have been used to model the potential critical hazard loading combinations. They are the Turkstra and Madsen rule (Turkstra and Madsen, 1980); Ferry-Borges-Castanheta model (Thoft-Christensen and Baker, 1982) and Wen's load coincidence method (Wen, 1981). As defined above, the basic idea of reliability design is to ensure that the reliability index β is at the same level for all possible hazard loading combinations.

Although reliability index provides a quantitative risk assessment, the weakness of such a commensurable criterion is its weak connection to the physical conditions associated with the bridge structural components at a given level of reliability index value. For structural safety, safety margin Z carries a random dispersion and according to the reliability formulation, higher uncertainty of z must require higher mean value for Z to research the same level of reliability. However, in view of the over-all safety of the entire bridge system, it is not clear if the reliability index is an accurate indicator since damage to certain components may not directly threaten the structural integration or collapse.

If bridge structural safety is a more desirable criterion, examination of criticality of the structural damage states that various hazard loadings may cause is another commensurable alternative. Jernigan and Hwang (1997) developed a list of damage states for different bridge elements. For the elements of expansion joints and bearings, possible damage states include: excessive displacement (inadequate seat width), excessive force. For the elements of columns, walls and footings, the damage states include: excessive column moment, footing moment, column shear, footing rotation and displacement; inadequacy of anchorage of longitudinal reinforcement, splice length in longitudinal reinforcement, transverse confinement steel in plastic zone. In terms of safety concerns, the damage states can be viewed as minor, moderate repairable and severe and safety threatening. A safety based commensurable standard can be developed for different level of damage severity by their potential safety concerns. The commensurable risk assessment then becomes a comprehensive damage scenario based risk analysis instead of reliability analysis.

A further extension of the safety concept is another current, actively pursued approach, the loss models based a commensurable criterion. In the following section, we present a life cycle cost based criterion, and its application in optimization of bridge design for multiple hazards. A more general application is seen in risk-based methodology for assessing seismic performance of highway systems (Stuart D. Werner, 2000). This approach aggregates the damage model with repairing cost model, traffic state model and social and economic loss model to evaluate the performance of bridge under the influences of various hazards.

In summary, to improve current AASHTO bridge design, various

commensurable criteria need to be examined. As discussed above, following each commensurable criterion, a quantitative risk assessment can be developed. However, different commensurable analyses are subjected to their technical limitations and uncertainties. A general methodology is needed to calibrate the valid engineering application scope for the commensurable analyses such as the acceptable scope of confidences, the accessible benefits for acceptance in multiple hazard bridge design, the acceptance or rejection gain or loss in view of the alternatives of risk aversion and risk mitigation, and real-data verifiability. Such a methodology should be suitable for existing safety design of major bridge components as well as for other desirable design considerations for construction, functionality, cost and maintenance, etc.

Expected Life-Cycle Cost for Multiple Hazards

Reliability based life-cycle cost design is one of the criteria recently proposed in performance-based design (Hiraishi,1998). This criteria can be incorporated with *optimization* design. This is generally referred to as level IV reliability based design. Recent developments in reliability-based optimization and applications to design of structural systems can be found in Frangopol and Corotis (1994). Several Federal Emergency Management Agency (FEMA) studies (FEMA ,1992a&b) dealt with decision making in rehabilitation of existing buildings. A standard benefit/cost model was developed for seismic rehabilitation of existing buildings. Field data in nine cities were collected to support the study. Ang and Leon (1996) studied optimal, cost-effective earthquake -resistance design criteria for reinforced concrete buildings in Mexico City and Tokyo. The target reliability was obtained and expected building damage cost was found to contribute the most in the total cost and the optimal design. Kanda and Ellingwood (1991) investigated optimal reliability-based design loads and load factors for possible implementation in a code format. Moreover, minimum building life-cycle cost design criteria were proposed by Wen and Kang (2001). In the following, we extend this approach into a reliability based optimal life-cycle cost design method for highway bridges under multiple hazards.

Over a time period t which may be the design life of a new structure or the remaining life of a retrofitted structure, the expected total cost can be expressed as a function of t and the design variable vector \mathbf{X} as follows (Wen and Kang, 2001):

$$E[C(t, \mathbf{X})] = C_1(\mathbf{X}) + E\left[\sum_{i=1}^{N(t)} \sum_{j=1}^k C_j e^{-\lambda t_i} P_{ij}(\mathbf{X}, t_i)\right] + \int_0^t C_m(\mathbf{X}) e^{-\lambda \tau} d\tau$$

where $E[.]$ is expected value; C_1 is initial cost for new or retrofitted facility; \mathbf{X} is design variable vector (design loads and resistance, or load and resistance factors associated with nominal design loads and resistance); i is severe loading occurrence

number, including joint occurrence of different hazards such as live, wind, and seismic loads; t_i is loading occurrence time, a random variable; $N(t)$ is total number of severe loading occurrences in t , a random variable; C_j is cost in present dollar value of j th limit state being reached at time of the loading occurrence, including costs of damage, repair, loss of service, and deaths and injuries; $e^{-\lambda t}$ is discounted factor of over time t ; λ is constant discount rate/year; P_{ij} is probability of j th limit states being exceeded given the i th occurrence of a single hazard or joint occurrence of different hazards; k is total number of limit states under consideration; C_m is operation and maintenance costs per year.

Implicit in the formula is the assumption that the structure will be restored to its original condition after each hazard occurrence. The discount factor $e^{-\lambda t}$ converts cost due to hazard that occurs in the future into present dollar value. Under the assumption that hazard occurrences can be modeled by a simple Poisson pulse process with occurrence rate of ν /year and for resistance that is time-invariant, assuming the limit state probabilities under a single or multiple hazards are time invariant and using the approximate method of load coincidence by Wen (1990), the expected total lifetime cost is given by

$$E[C(t, X)] = C_I + C_F \frac{(1 - e^{-\lambda t})}{\lambda} + C_m \frac{(1 - e^{-\lambda t})}{\lambda}$$

where C_F is total expected cost due to all (k) limit states. Each limit state can be caused by a single hazard or the joint occurrence of more than one hazard.

$$C_F = \sum_{l=1}^k C_l \left[\sum_{i=1}^n \nu_i P_l^i + \sum_{i=1}^{n-1} \sum_{j=i+1}^n \nu_{ij} P_l^{ij} + \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^n \nu_{ijk} P_l^{ijk} + \dots \right]$$

where P_l^i is probability of limit state l given the occurrence of hazard i ; P_l^{ij} is probability of limit-state l given the coincidence of hazards i and j ; P_l^{ijk} is probability of limit-state l given the joint occurrence of hazards i , j , and k ; μ_{d_i} is the mean duration of hazard i . ν_i is mean occurrence rate of hazard i ;

$$\nu_{ij} = \nu_i \nu_j (\mu_{d_i} + \mu_{d_j})$$

is the coincidence rate of hazards i and j ;

$$\nu_{ijk} = \nu_i \nu_j \nu_k (\mu_{d_i} \mu_{d_j} + \mu_{d_j} \mu_{d_k} + \mu_{d_i} \mu_{d_k})$$

is the coincidence rate of hazards i , j , and k .

It is assumed that the structure is restored to its original condition if damaged during a hazard occurrence.

Design Optimization Based on the Cost Minimization Criteria

If the objective of a bridge design for multiple hazards is to minimize the total expected life-cycle cost, i.e., balance between the initial cost and the expected failure (limit state) costs, then the optimization problem described above is an unconstrained minimization. Proper constraints may be introduced in the above minimization; the constraints may be the limits of design variables or minimum acceptable reliability levels for limit states, or both. Parametric studies of optimal design under a single or multiple hazards are carried out in the following section.

Parametric Study

- For Single Hazard

Under a single hazard modeled by a Poisson pulse process with an occurrence rate of ν /year and for a resistance that is time-invariant, a parametric study has been carried out for the optimal design against wind or seismic hazard. In this study, the following assumptions are made: (1) Hazard intensity is modeled by a normal distribution; (2) Bridge resistance has a normal distribution and a single limit state of design intensity being exceeded is considered. (3) Initial cost and failure cost are considered. The maintenance cost is not considered.

The expected total cost as a function of the lifetime t can be expressed as

$$E[C(t, Y)] = C_1(Y) + \nu \Phi \left(-\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \right) \frac{C}{\lambda} (1 - e^{-\lambda t})$$

where $C_1 = C_0 \left[1 + k \left(\frac{\mu_R}{\mu_S} - 1 \right) \right]$ and $C = gC_0$, in which C_0 is the initial cost at

$\mu_R = \mu_S$, k and g are normalized cost ratios.

The optimal solution can be determined from the above equation by minimization. Figure 1 and Figure 2 show the optimal reliability index under wind hazard alone and seismic hazard alone for three cost levels.

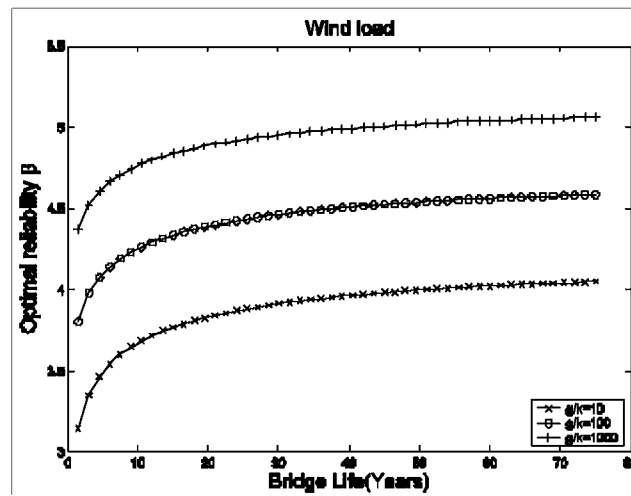


Figure 1. Optimal reliability index under wind hazard alone .

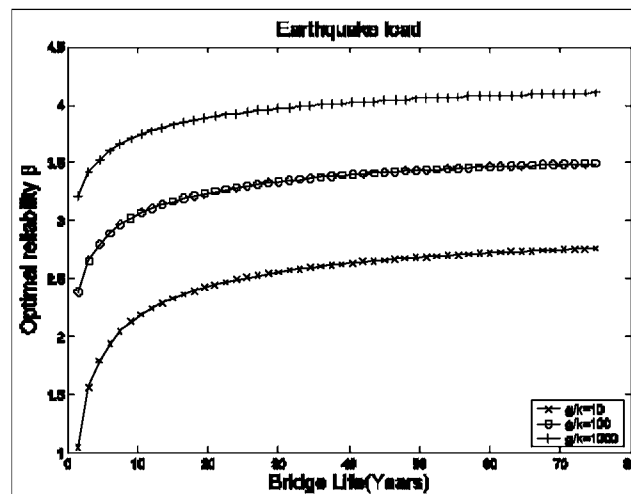


Figure 2. Optimal reliability index under seismic hazard alone .

- For two hazards

Similarly, under the same assumptions as those in the single hazard situation, the expected total cost for bridges under two hazards as a function of the lifetime t can be expressed as

$$E[C(t, Y)] = C_1(Y) + \left\{ v_1 \Phi \left(-\frac{\mu_R - \mu_{S_1}}{\sqrt{\sigma_R^2 + \sigma_{S_1}^2}} \right) + v_2 \Phi \left(-\frac{\mu_R - \mu_{S_2}}{\sqrt{\sigma_R^2 + \sigma_{S_2}^2}} \right) \right. \\ \left. + v_1 v_2 (\mu_{d_1} + \mu_{d_2}) \Phi \left[-\frac{\mu_R - (\mu_{S_1} + \mu_{S_2})}{\sqrt{\sigma_R^2 + \sigma_{S_1}^2 + \sigma_{S_2}^2}} \right] \right\} \frac{C}{\lambda} (1 - e^{-\lambda t})$$

The optimal solution can be found from the above equation by numerical minimization. Figure 3 shows the optimal reliability index under combined wind and seismic hazards for three cost levels.

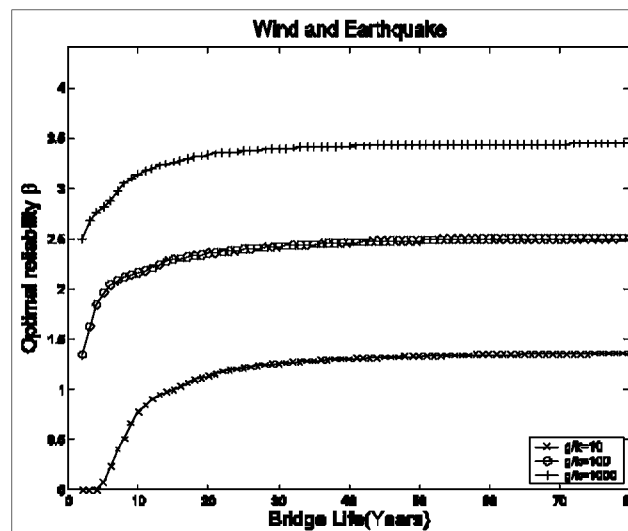


Figure 3. Optimal reliability index under combined wind and seismic hazards.

Incomplete Information and Statistically Unpredictable Events

Risky events and uncertain events are clearly distinct concepts. The distinction is that in a risky event the uncontrollable randomness comes from a known probability distribution, whereas in an uncertain event the probability distribution is unknown.

In the previous section, although the discussions are all focused on the risky events, our knowledge about the event probability distribution is not perfect, and in most cases the probability distributions are empirical or even hypothetical. Such a condition is tolerable so long as we have verifiable key statistical properties such as the mean and standard deviation or even just the distribution consistency. Moses and Ghosn (1985) found that “safe existing designs” are relatively insensitive to errors in

the statistical data base as long as the same statistical data and criteria are used to find the target reliability index. This is due to the reason that a change in a statistical property will influence the average β value, which only changes the calibration reference with respect to a cross-board new average.

The tolerance of information incompleteness may be extended to functions that are lack of sufficient statistical data. In such a situation, Monte Carlo simulations are suitable to create a large number of “experiments” through the random generation of sets of corresponding input variables of the function. Then the probability distribution of the function values can be estimated.

While the unintentional hazards can be modeled by stochastic processes, which possess ergodic statistical properties, there is also a different type of hazards, intentional malicious attacks such as the 9/11 terrorist attack and Oklahoma federal government building explosion. Such hazards are typical uncertain events that the predictability of a stochastic model is invalid since the behaviors of the terrorists no longer follow any invariant statistical distributions. The approach for design of highway bridges to resist terrorist attacks becomes a rather involved process of strategic planning and decision making and engineering of protective solutions. The complexity may be illustrated through the following example. Consider a simple “ordinal” strategy game based on Revealed Preference Theory (RPT) by Samuelson, (1997). Assume that the objective of selecting strategy paths by the two opponents is to maximize their own payoffs, and the outcomes only matter in order, magnitude is irrelevant. For a bridge system, the defender has two choices of increasing the security and preventative measures vs. not taking any of such actions. The offender has also two choices of selecting the bridge as the attack target vs. giving-up the attack.

The following matrix represents the outcomes of all possible strategy selections, where the payoff preferences are such that $a \gg b \gg c \gg d$ for both opponents.

	Increase security	Without increase security
Attack	(c, d)	(d, a)
Give-up Attack	(b, c)	(a, b)

In the above matrix, the pair of symbols in the parenthesis is the corresponding payoff in which the first component represents the payoff for the defender, the second is for the offender.

For the defender, the meanings of the payoffs are explained below:
 “a” can be interpreted as without increased security and the bridge is not attacked.

“b” can be interpreted as with increased security and the bridge is not attacked.
“c” can be interpreted as with increased security and the bridge is still attacked.
“d” can be interpreted as without increased security and the bridge is attacked.
For the offender, the corresponding meanings of the payoffs are explained below:
“a” can be interpreted as attacking the bridge with chance to bring damage.
“b” can be interpreted as giving-up a target that is possible to bring damage to.
“c” can be interpreted as giving-up a hard target
“d” can be interpreted as attacking a hard target with little chance to bring damage.

Assume the defender moves first and has to reveal his choice. What is the best strategy for the defender? The answer has to be increase the security. Since the outcome “d” is unacceptable, and making a choice of not increasing the security first will prompt for the opponent to select attack for his best payoff of “a”, which corresponds to outcome “d” for the defender.

The above simple example describes a case of one type of attack vs. one protection strategy. It indicates that toughness to penetrate a bridge security is in reverse proportion to the chance of the bridge being attacked. Therefore, the probability of terrorist attack is adaptive. This aspect is completely different from the unintentional hazards in that the occurrences of the hazards are independent of the structures’ hazard resilience.

We should point out that the reality is more complicated than the simple example can illustrate. Currently in the US, there are several hundred thousands public bridges, any security improvement for this inventory is a significant resource demand and increase of security is not necessarily a guarantee of successful prevention of terrorist attacks since there are many different means that terrorists may choose. Furthermore, a single bridge protection is merely altering the terrorists target to somewhere else.

For bridge design against purposely plotted malicious attacks such as explosion and intentional collision and accident events, the hazard nature is statistically unpredictable, therefore, the ordinary random stochastic model is no longer valid. New model such as the one given above must be developed in that it typically has to consider various uncertainties such as intent, method of attack, possible consequences, and difficulty of penetration. Even then, the design philosophy may be different. One possible approach is to explore some critical scenario based design. The target of such effort will need to integrate planning, hazard prevention, instance response and disaster preparedness.

Summary

In considering natural and man-made catastrophes for future highway bridges, several challenges have to be surmounted. First, different uncertainties, which cannot be modeled in the same approach, for example, earthquake is reasonably modeled as an independent stochastic process, but intentional man-made is not suitable for such a model. Second, design load factors for different catastrophes need to consider in a wide-spectrum of issues: likelihood vs. damage cost, mitigation approaches (e.g. protection technology, retrofit, routing) vs. consequences. Third, overall design objectives achievable under current practical constraints should be evaluated and failure scenarios beyond the design resist capacity should be modeled. In general, the approach to take in dealing with natural and man-made catastrophes has to be integrated protection, mitigation, response and preparedness.

Acknowledgement

This paper is a progress report of a research project funded by the US Federal Highway Administration to develop a framework of multiple hazard design for highway bridges.

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