

# DISTRIBUTION OF DAMAGE RATING OF BRIDGE MEMBERS AND A FEW CONSIDERATIONS

Hiroshi Sato<sup>1</sup>

## Abstract

In this paper, distributions of damage rating of bridge members were studied and described. According to the trend of distributions and their change with age, three types of deterioration were introduced.

Deteriorations of some damages of bridge members were modeled by Markov process, and transition probability matrices were calculated. The predicted relative frequency distributions agreed fairly well with the inspection results.

## Introduction

Japan has improved highways and highway bridges. The highway stocks have increased in volume. There are one hundred and fifty thousands of national highway bridges longer than 15 m. Cost-effective and systematic bridge management is required under such situation.

Several years ago, the author and his colleagues proposed Bridge Management System[1], [2]. The BMS consisted of two program modules, which were the condition evaluating module and the rehabilitation planning module. The condition evaluating module evaluated the present bridge condition from the rating of bridge members. The rehabilitation planning module proposed rehabilitation plan which would minimize the Life Cycle Cost of bridges. The benefit B of one rehabilitation plan was defined as LCC saved by the repair work, and it was calculated by following equations,

$$B = C_0 - C_1 \quad (1)$$

$$C_0 = A \left[ \frac{(1+i)^T}{(1+i)^T - 1} \right] \left[ \frac{1}{(1+i)^r} \right] \quad (2)$$

$$C_1 = A \left[ \frac{(1+i)^T}{(1+i)^T - 1} \right] \left[ \frac{1}{(1+i)^{r+e}} \right] \quad (3)$$

Here,  $C_0$  is present value of rehabilitation cost in the future under the condition that no rehabilitation is performed at present and the bridge will be replaced  $tr$  year from now, and will be replaced at  $T$  year interval.  $C_1$  is present value of rehabilitation cost in the future

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<sup>1</sup> Professor, Graduate School of Systems and Information Engineering, University of Tsukuba

under the condition that the bridge is rehabilitated at present, and it will be replaced ( $tr+e$ ) year from now, and will be replaced at  $T$  year interval.  $A$  is the replacement cost, and  $i$  is the discount rate. Rehabilitation will extend bridge life by  $e$  years as is shown in Fig.1. In the figure,  $T$ ,  $tr$  and  $e$  are explained along with Standard Deterioration Curve. It is clear from the figure that deterioration characteristics are necessary to calculate the benefit defined by Equation (1). At that time, however, we did not have enough data to predict deterioration of bridges or bridge members.

Ministry of Land, Infrastructure, Transport, and Tourism issued the Periodical Inspection Manual for Bridges (Draft) [3] in 2004. Before this Manual, the Inspection Manual for Bridges (Draft) [4] issued by Public Works Research Institute in 1988 was used for inspection of highway bridges. Now most of the national highway bridges are being inspected according to the new Manual. Data on deterioration of highway bridge member are being accumulated. On the other hand, modeling of deterioration processes have been studied by many researchers, and Markov process models are sometimes adopted to Bridge Management Systems including PONTIS.

In this paper, distributions of damage rating of bridge members are shown first with their age. According to trend of the distributions and their change with age, three types of deterioration are introduced. Deteriorations of some damages of bridge members are assumed to be Markov process, and their transition probability matrices are calculated.

### **Distribution of Damage Rating of Bridge Members**

According to the Periodical Inspection Manual for Bridges (Draft), damage rating of bridge members should be given as follows:

TABLE 1 DAMAGE RATING OF BRIDGE MEMBERS [3]

Damage Rating	State of Damage, Action Required
A	No damage, or the damage is so light that repair is unnecessary.
B	Repair is necessary according to the situations.
C	Prompt repair or other work is necessary.
E1	Emergency response is necessary to keep safety of the bridge structure.
E2	Emergency response is necessary from the other reasons.
M	Maintenance work is necessary.
S	Detailed survey is necessary

Kinds of damages to be inspected are specified according to the member and its material [3] [5]. For example, corrosion, cracking, looseness/falling, rupture, deterioration of corrosion protection, gap, anchors, noise/ vibration, deflection, and deformation should be inspected in case of steel main girders. In case of concrete deck, cracking, scaling/

exposure of reinforcement, leakage/ free lime, falling, damaged reinforcement, separation, gap, anchors, change of color, stagnant water, noise/ vibration, deflection, and deformation should be inspected.

Damage ratings of bridge members were evaluated and their distributions were plotted with their age by the Ministry of Land, Infrastructure, Transport, and Tourism. Some of the results are shown in the Figures 2.1-2.4.

According to the Fig. 2.1, damage rating of corrosion of steel main girders seems to change to higher level with their age. Similar trend can be seen in the Fig. 2.2, which shows distribution of scaling/ exposure of reinforcement of concrete floor deck. The trend that the damage rating changes to higher level with their age seems quite natural. On the other hand, however, trend of damage rating does not seem to become severe with age in case of cracking of concrete floor deck (Fig. 2.3). This trend might come from the deterioration characteristics of cracking of concrete floor deck or maintenance/rehabilitation activities to the cracking. At present, the reason is not clear. In case of cracking of steel main girders (Fig. 2.4), damage rating other than A is scarcely observed. For this type of damages, it will be very difficult to predict when and where the damage will take place.

The three types of deterioration are summarized as follows:

Type 1: Damage rating changes to higher level with age. (for example, corrosion of steel main girders (Fig. 2.1), and scaling/ exposure of reinforcement of concrete floor deck (Fig. 2.2))

Type 2: Damage rating does not change to higher level with age. (for example, cracking of concrete floor deck (Fig. 2.3))

Type 3: Damage rating other than A is scarcely observed. (for example, cracking of steel main girders (Fig. 2.4))

### **Application of Markov Chain Model to Prediction of Damage Rating**

If a deterioration process of a bridge member is assumed to be a Markov process, and if the transition probability matrix between two deterioration states is assumed to be steady, then the state probability can be predicted as follows.

$$\boldsymbol{\pi}(n) = \boldsymbol{\pi}(n-1)\mathbf{P} = \boldsymbol{\pi}(0)\mathbf{P}^n \quad (4)$$

where  $\boldsymbol{\pi}(n)$  : state probability vector at time n

$$\boldsymbol{\pi}(n) = [q_1(n) \quad q_2(n) \quad \cdots \quad \cdots \quad q_m(n)]$$

$\mathbf{P}$  : transition probability matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & \cdots & p_{mm} \end{bmatrix}$$

$p_{ij}$  : transition probability from state i to state j

If maintenance cost for a state for one time is assumed, expected cumulative maintenance cost up to the time n can be predicted as follows.

$$Ct = \sum_i \pi(i)C = \sum_i \pi(0)\mathbf{P}^i \mathbf{C} \quad (5)$$

where  $\mathbf{C}$  : maintenance cost vector for one time

$$\mathbf{C} = [c_1 \quad c_2 \quad \cdots \quad \cdots \quad c_m]^T$$

In general, transition probability can be calculated from transition data. In case of deterioration of bridge members, transition data can be obtained from two consecutive inspection results of the same member of the same bridge. Unfortunately, the second inspection results according to the Bridge Inspection Manual 2004 have not been available yet because the inspection interval is 5 years. Therefore transition probability was estimated from state probability vector so that prediction error may be minimum. The state probability was assumed to be the same as the observed relative frequency of ratings shown in the Fig.2. The methods [6] are outlined as follows.

$$\text{prediction error } \varepsilon \equiv \sum_{t=1}^T \sum_{j=1}^s (y_{tj} - \sum_{i=1}^s y_{t-1,i} p_{ij})^2 \rightarrow \text{minimum} \quad (6)$$

on the condition that

$$p_{kh} = 0 \quad (k,h) \in J \quad (7)$$

$$\sum_{j=1}^s p_{ij} = 1, \quad i = 1, 2, \dots, s \quad (8)$$

where  $\mathbf{P}$  : transition probability matrix

$\mathbf{y}_t$  : state probability vector at time t

Applying the method of Lagrange undetermined multipliers,

$$\frac{\partial}{\partial p_{ij}} \left[ \sum_{t=1}^T \sum_{j=1}^s (y_{tj} - \sum_{i=1}^s y_{t-1,i} p_{ij})^2 + \sum_{i=1}^s \lambda_i (\sum_{j=1}^s p_{ij} - 1) + \sum_{(k,h) \in J} \mu_{kh} p_{kh} \right] = 0 \quad (9)$$

$$\sum_{t=1}^T y_{t-1,i} y_{tj} - \sum_{k=1}^s \sum_{t=1}^T (y_{t-1,i} y_{t-1,k}) p_{kj} = \lambda_i + \mu_{ij} \quad (\mu_{ij} = 0 \text{ when } (i,j) \notin J) \quad (10)$$

Equation (10) can be rewritten into

$$\mathbf{Y} - \mathbf{Z}\mathbf{P} = \lambda\xi^T + \mathbf{U} \quad (11)$$

$$\text{where } [\mathbf{Y}]_{ij} \equiv \sum_{t=1}^T y_{t-1,i} y_{tj}$$

$$[\mathbf{Z}]_{ij} \equiv \sum_{t=1}^T y_{t-1,i} y_{t-1,j}$$

$$[\lambda]_i \equiv \lambda_i$$

$$[\mathbf{U}]_{ij} \equiv \mu_{ij}$$

$$[\xi]_i \equiv 1$$

At first,  $\mu_{ij}$  will be calculated from the following equations.

$$\mathbf{P} = \mathbf{Z}^{-1}(\mathbf{Y} + \frac{1}{s}\mathbf{U}\xi\xi^T - \mathbf{U}) \quad (12)$$

$$p_{kh} = 0 \quad (k,h) \in J$$

Then,  $\mathbf{P}$  can be calculated by substituting the  $\mu_{ij}$  into the equation (12).

Using the above method, the transition probability matrices were estimated and relative frequency distributions were calculated for corrosion of steel main girders, scaling/ exposure of reinforcement of concrete floor deck, and cracking of concrete floor deck. The results are shown in the Figures 3.1-3.3.

Since the frequencies other than A,B and C were very few, only the three states were considered in the calculation. The predicted relative frequency distributions (C. in the figures) agree fairly well with the inspection results (A in the figures). The prediction error defined as in Equation (6) is also shown at the caption of C. As is clear from Equation (9), the calculated transition probability matrices automatically satisfy the Equation (8), however, the calculated transition probability does not necessarily take value between 0 and 1. When negative value was found, that transition probability was assumed 0, then the transition probability matrices were calculated again [6].

Transition probabilities from B to A for corrosion of steel main girders, and cracking of concrete floor deck are not negligibly small. If no maintenance/rehabilitation work is done, lower left components of the matrices should be equal to 0. The reason of these transition probabilities from B to A is not clear at present.

## **Conclusions**

In this paper, distributions of damage rating of bridge members were studied. As for corrosion of steel main girders, and scaling/ exposure of reinforcement of concrete floor deck, the change of their distribution with age showed natural trends, namely damage rating changed to higher level with age. But not all the damages showed this trend. For example, damage rating did not change to higher level with age in case of cracking of concrete floor deck. In case of cracking of steel main girders, damage rating other than A was scarcely observed.

Deteriorations of some damages of bridge members were modeled by Markov process, and transition probability matrices were calculated for corrosion of steel main girders, scaling/ exposure of reinforcement of concrete floor deck, and cracking of concrete floor deck. The predicted relative frequency distributions agreed fairly well with the inspection results.

## **Acknowledgments**

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## **References**

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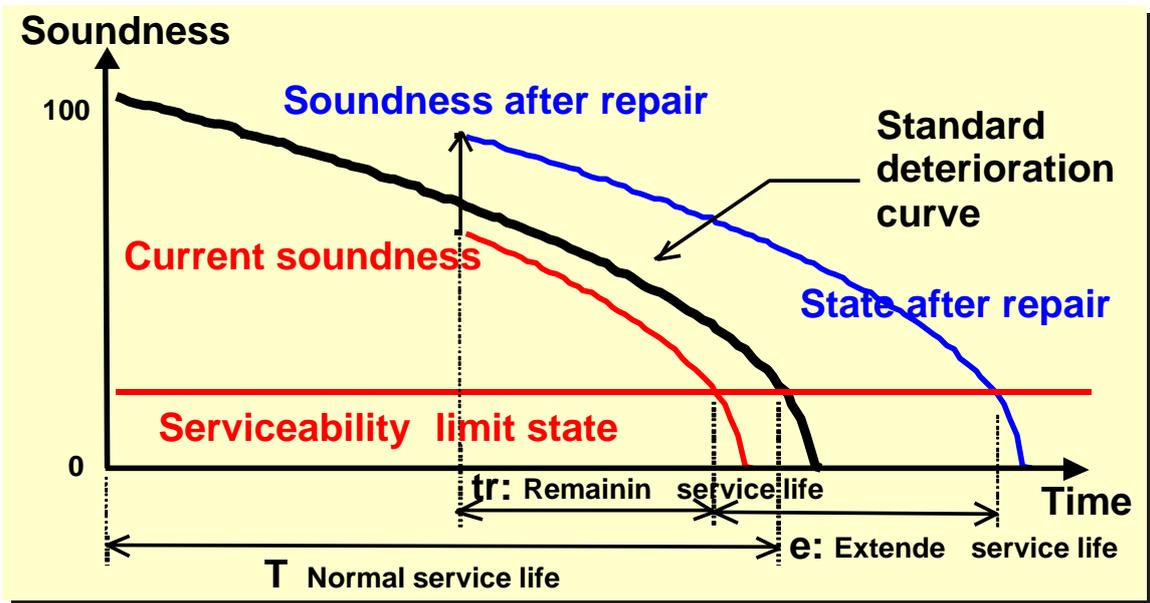


FIG.1 EFFECTS OF REPAIR

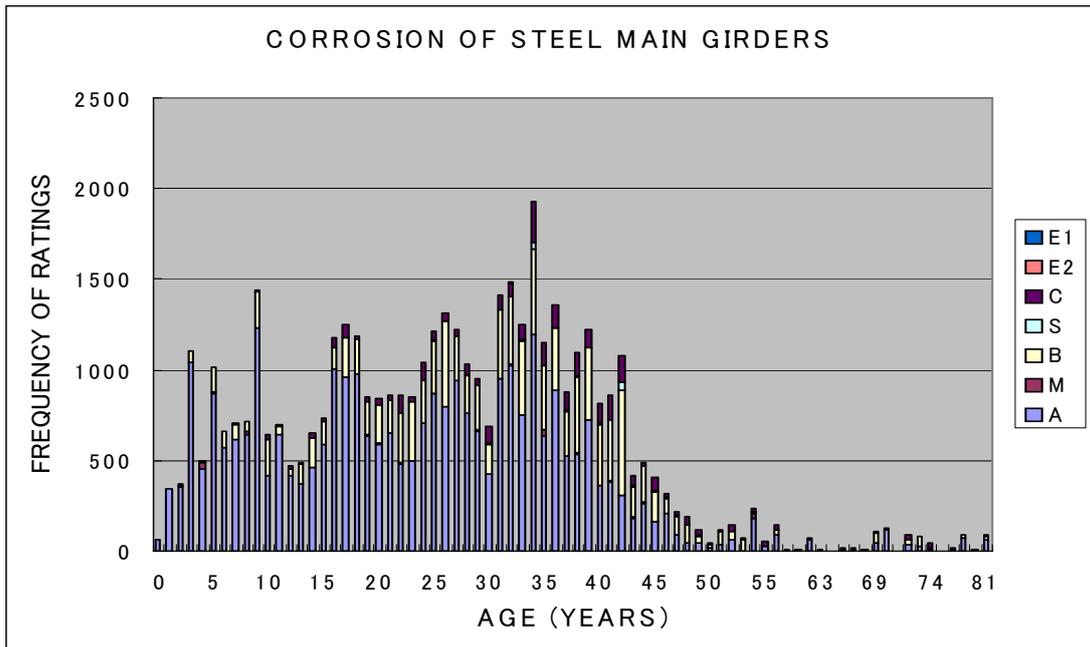
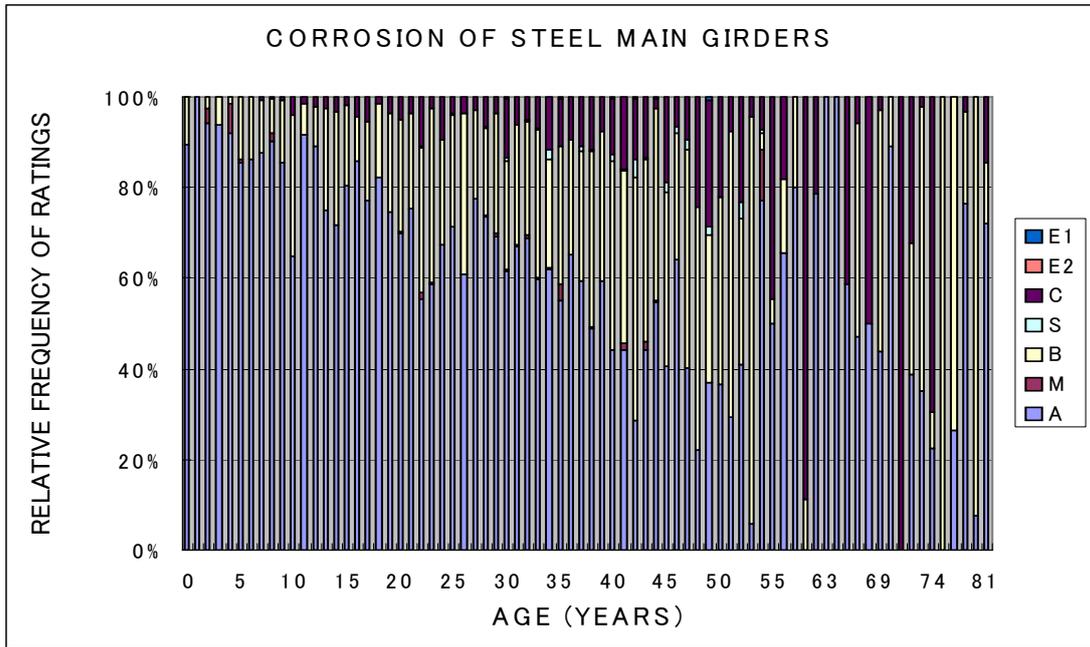


FIG. 2.1 DISTRIBUTION OF DAMAGE RATING FOR CORROSION OF STEEL MAIN GIRDERS (SOURCE: MINISTRY OF LAND, INFRASTRUCTURE, TRANSPORT, AND TOURISM)

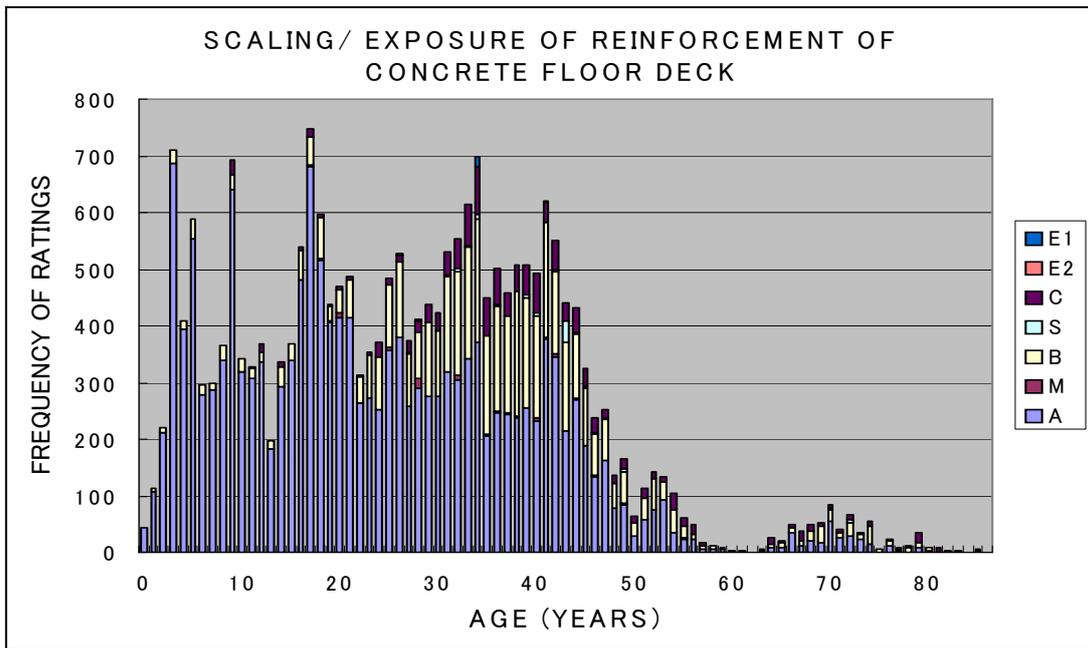
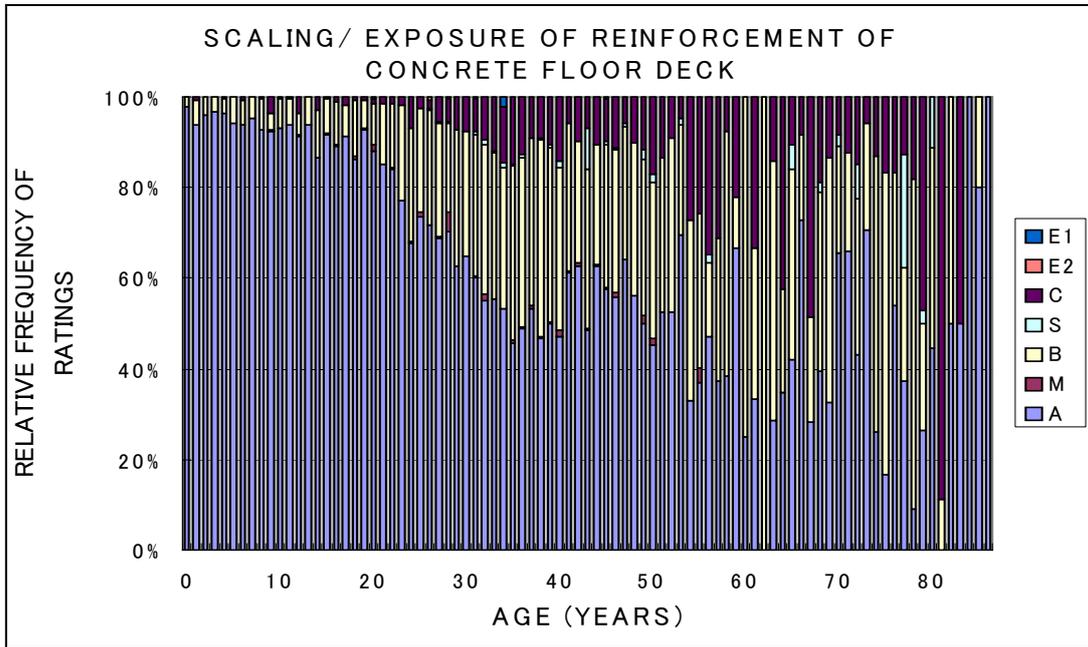


FIG. 2.2 DISTRIBUTION OF DAMAGE RATING FOR SCALING/ EXPOSURE OF REINFORCEMENT OF CONCRETE FLOOR DECK (SOURCE: MINISTRY OF LAND, INFRASTRUCTURE, TRANSPORT, AND TOURISM)

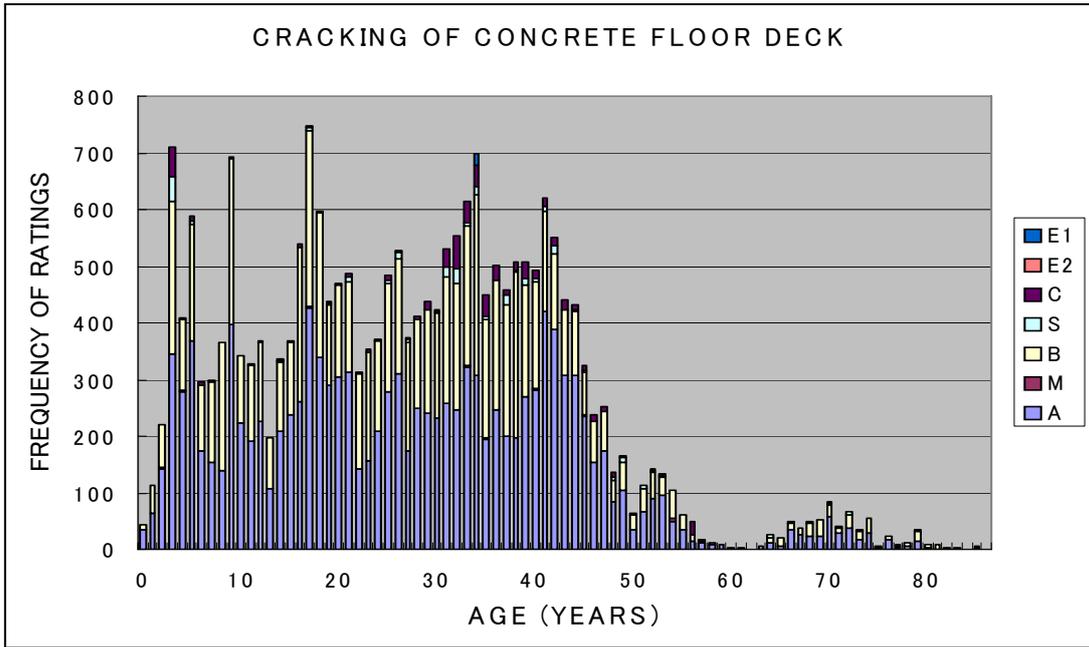
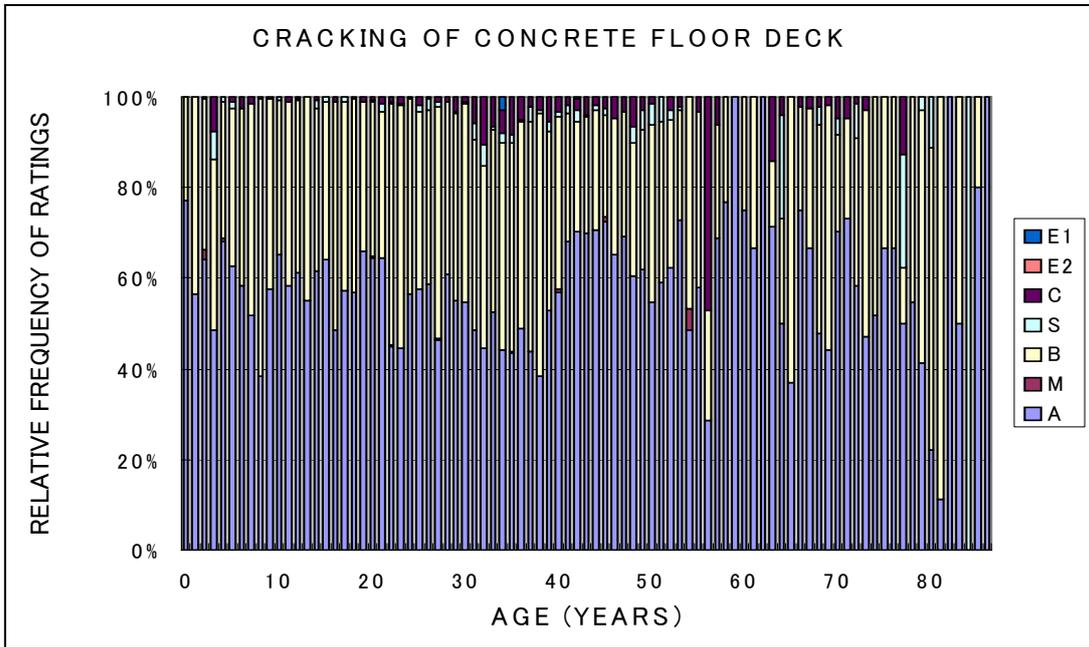


FIG. 2.3 DISTRIBUTION OF DAMAGE RATING FOR CRACKING OF CONCRETE FLOOR DECK (SOURCE: MINISTRY OF LAND, INFRASTRUCTURE, TRANSPORT, AND TOURISM)

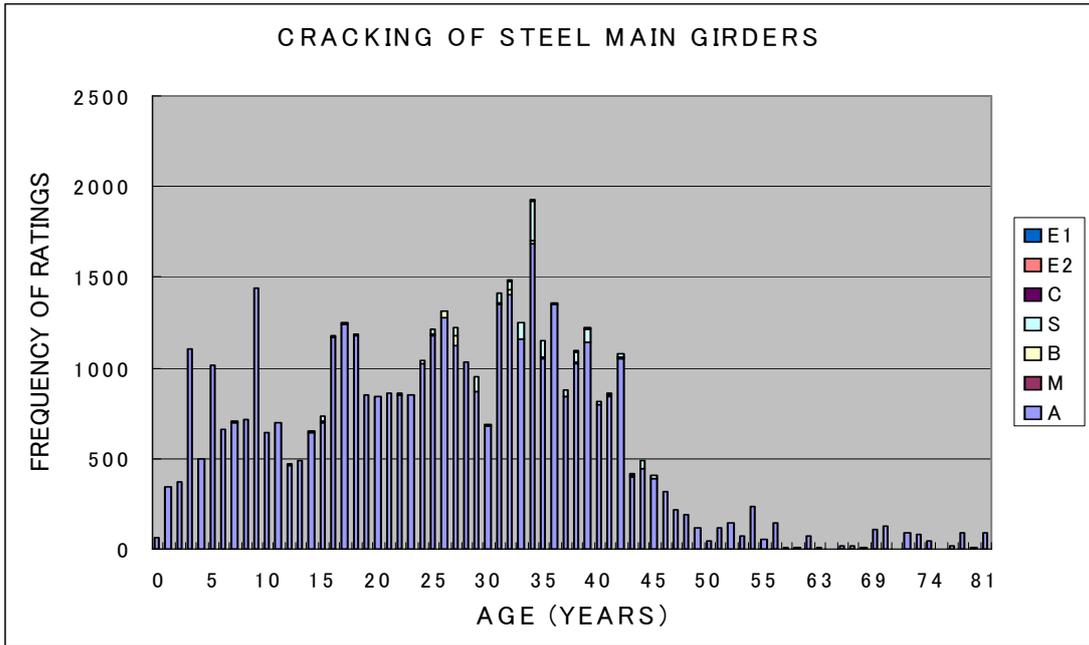
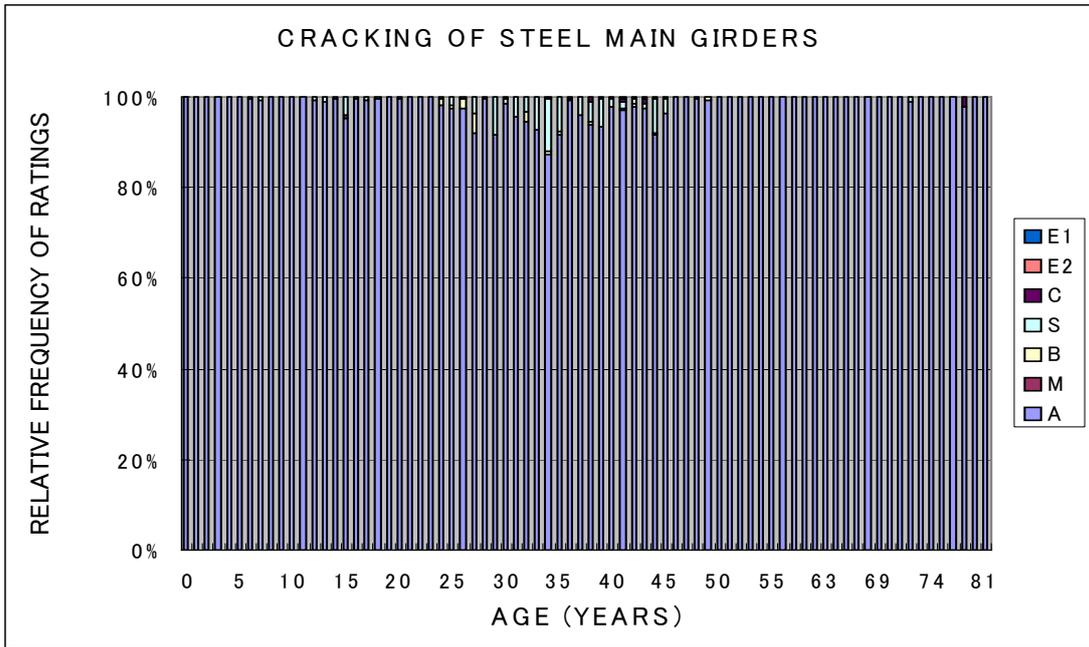
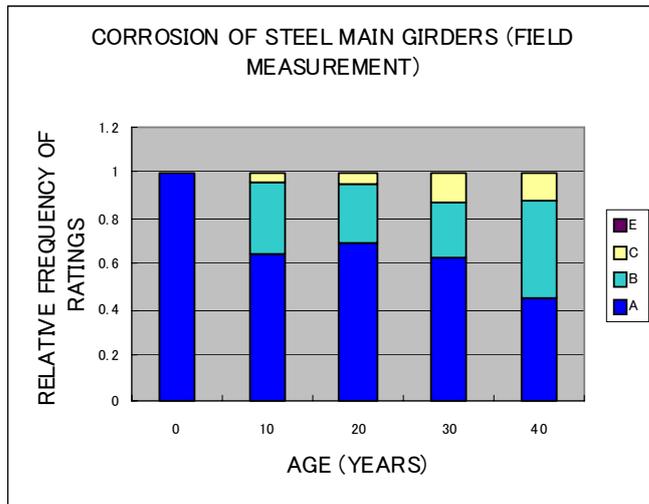


FIG. 2.4 DISTRIBUTION OF DAMAGE RATING FOR CRACKING OF STEEL MAIN GIRDERS (SOURCE: MINISTRY OF LAND, INFRASTRUCTURE, TRANSPORT, AND TOURISM)



A. RESULT OF INSPECTION

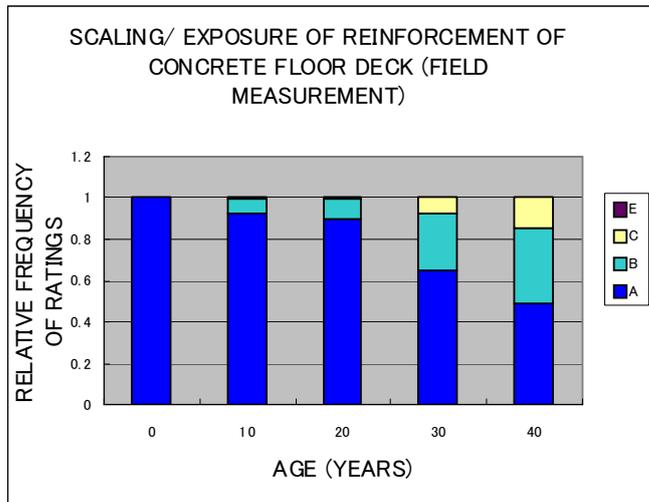
B. ESTIMATED TRANSITION PROBABILITY MATRIX

	A	B	C
A	0.638	0.310	0.051
B	0.673	0.086	0.240
C	0	1	0



C. PREDICTION FROM THE TRANSITION PROBABILITY MATRIX (PREDICTION ERROR=0.034)

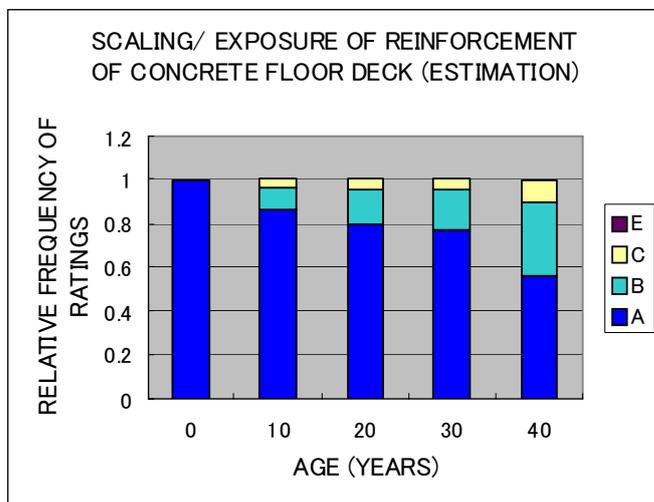
FIG. 3.1 DISTRIBUTION OF DAMAGE RATING FOR CORROSION OF STEEL MAIN GIRDERS



A. RESULT OF INSPECTION

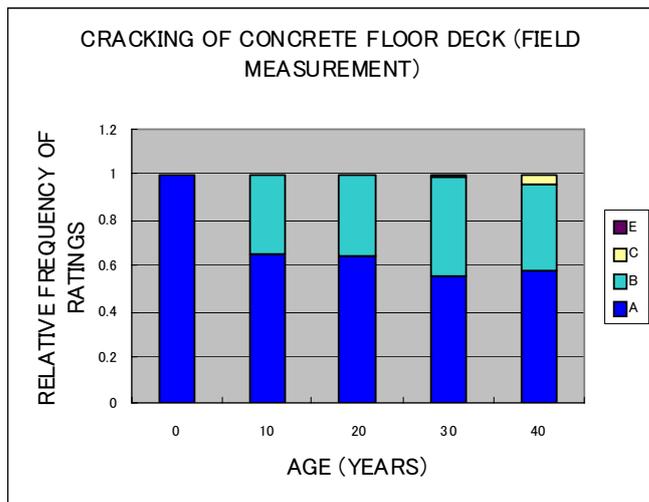
B. ESTIMATED TRANSITION PROBABILITY MATRIX

	A	B	C
A	0.861	0.104	0.035
B	0.000	0.981	0.018
C	0	0	1



C. PREDICTION FROM THE TRANSITION PROBABILITY MATRIX (PREDICTION ERROR=0.056)

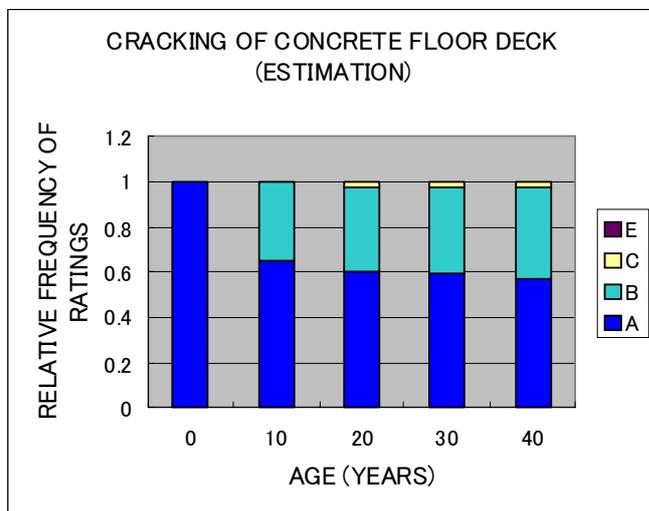
FIG. 3.2 DISTRIBUTION OF DAMAGE RATING FOR SCALING/ EXPOSURE OF REINFORCEMENT OF CONCRETE FLOOR DECK



A. RESULT OF INSPECTION

B. ESTIMATED TRANSITION PROBABILITY MATRIX

	A	B	C
A	0.654	0.346	0
B	0.493	0.452	0.055
C	0	1	0



C. PREDICTION FROM THE TRANSITION PROBABILITY MATRIX (PREDICTION ERROR=0.009)

FIG. 3.3 DISTRIBUTION OF DAMAGE RATING FOR CRACKING OF CONCRETE FLOOR DECK