TRANSITION PROBABILITY MATRIX OF BRIDGE MEMBERS' DAMAGE RATING

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Abstract

Bridge members' damage characteristics were studied using the inspection records. Damages can be classified into three types. It was found that most of the damages were classified into Type 3, where damage rating other than A was scarcely observed.

The transition probability matrices were estimated for all the bridge members' damages except for the Type 3, using the damage rating records of the bridges up to the age of 40 years. The transition probability was determined so that prediction error may be minimum. The relative frequency distributions predicted from the transition probability matrices agreed fairly well with the inspection results. The transition probability p_{BA} of Type 2, where damage rating does not change to higher level with age, was larger than that of Type 1, where damage rating changes to higher level with age.

The transition probability matrices depend on the information on the repair or rehabilitation conducted to the bridge. Since the information was not available for the data we analyzed, we cannot determine the lower left components of the matrix. The problem will be solved by analyzing transition data obtained from two consecutive inspection results of the same member of the same bridge, which was not repaired or rehabilitated between the two inspections.

Introduction

There are one hundred and fifty thousands of highway bridges longer than 15 m in Japan. The highway stocks have increased in volume. Cost-effective and systematic bridge management is required under such situation. Ministry of Land, Infrastructure, Transport, and Tourism issued the Periodical Inspection Manual for Bridges (Draft) [1] in 2004. Now most of the national highway bridges are being inspected according to the Manual. Data on deterioration of highway bridge member are being accumulated. On the other hand, modeling of deterioration processes have been studied by many researchers, and Markov process models are sometimes adopted to Bridge Management Systems including PONTIS.

In the previous paper [2], distributions of damage rating of bridge members were studied. As for corrosion of steel main girders, and spalling/ exposure of reinforcement of concrete deck, the change of their distribution with age showed natural trends, namely

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damage rating changed to higher level with age. But not all the damages showed this trend. For example, damage rating did not change to higher level with age in case of concrete deck crack. In case of crack of steel main girders, damage rating other than A was scarcely observed. Deteriorations of some damages of bridge members were modeled by Markov process, and transition probability matrices were calculated for corrosion of steel main girders, spalling / exposure of reinforcement of concrete deck, and concrete deck crack.

In this paper, transition probability matrices were presented for all the bridge members' damages except for damages whose rating other than A was scarcely observed. The relative frequency distributions predicted from the transition probability matrices were compared with the inspection results. Several calculation methods of transition probability matrices were compared and discussed.

Types of Damage Rating of Bridge Members

According to the Periodical Inspection Manual for Bridges (Draft), damage rating of bridge members should be given as follows:

TABLE 1 DAMAGE RATING OF BRIDGE MEMBERS [1]

Damage Rating	State of Damage, Action Required
A	No damage, or the damage is so light that repair is unnecessary.
B	Repair is necessary according to the situations.
C	Prompt repair or other work is necessary.
E1	Emergency response is necessary to keep safety of the bridge structure.
E2	Emergency response is necessary from the other reasons.
M	Maintenance work is necessary.
S	Detailed survey is necessary

Kinds of damages to be inspected are specified according to the member and its material [1] as is shown in Table 2.

In the previous paper [2], damages were classified into three types according to their deterioration characteristics:

Type 1: Damage rating changes to higher level with age. (For example, corrosion of steel main girders, and spalling/ exposure of reinforcement of concrete deck.)

Type 2: Damage rating does not change to higher level with age. (For example, crack of concrete deck.)

Type 3: Damage rating other than A is scarcely observed. (For example, crack of steel main girders.)

Based on the inspection records, damages were classified and shown in the Table 2, where damages of Type 1 and Type 2 are colored by yellow. It was found that most of the damages were classified into Type 3. The transition probability matrices were calculated for the damages of Type 1 and Type 2. For the damages of Type 3, transition probability matrices were not calculated, but identity matrices seem appropriate for their transition probability matrices.

Prediction of Transition Probability Matrix

If a deterioration process of a bridge member is assumed to be a Markov process, and if the transition probability matrix is assumed to be homogeneous, then the state probability can be predicted as follows.

$$\boldsymbol{\pi}(n) = \boldsymbol{\pi}(n-1)\mathbf{P} = \boldsymbol{\pi}(0)\mathbf{P}^{n} \qquad (1)$$

where $\boldsymbol{\pi}(n)$: state probability vector at time n

$$\boldsymbol{\pi}(n) = \begin{bmatrix} q_1(n) & q_2(n) & \cdots & \cdots & q_m(n) \end{bmatrix}$$

$q_i(n)$: probability of state i at time n

$\mathbf{P}$: transition probability matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & \cdots & p_{mm} \end{bmatrix}$$

p_{ij} : transition probability from state i to state j

In case of deterioration of bridge members, transition data can be obtained from two consecutive inspection results of the same member of the same bridge. Unfortunately, the second inspection results according to the Bridge Inspection Manual 2004 were not available at the time of our analysis. Therefore transition probability was estimated from the state probability vectors of the bridges up to the age of 40 years. The time for one-step was one-year. The state probability was assumed to be the same as the observed relative frequency of ratings. The transition probability was determined so that prediction error may be minimum. The methods [3] are outlined as follows.

$$\mathit{prediction\ error}\ \varepsilon \equiv \sum_{t=1}^{T}\sum_{j=1}^{s}(y_{tj}-\hat{y}_{tj})^2 \rightarrow \text{minimum} \qquad (2)$$

where y_{tj} : observed probability of state j at time t

$\hat{y}_{tj}$: predicted probability of state j at time t

$$\hat{y}_{tj} = \sum_{i=1}^{s} y_{t-1,i}\, p_{ij} \qquad (3)$$

on the condition that

$$p_{kh} = 0 \quad (k,h) \in J \tag{4}$$

$$\sum_{j=1}^{s} p_{ij} = 1, \quad i = 1,2,\cdots s \tag{5}$$

where $\mathbf{P}$: transition probability matrix

$\mathbf{y_t}$: state probability vector at time t

Applying the method of Lagrange undetermined multipliers,

$$\frac{\partial}{\partial p_{ij}}[\sum_{t=1}^{T}\sum_{j=1}^{s}(y_{tj} - \sum_{i=1}^{s} y_{t-1,i}\, p_{ij})^2 + \sum_{i=1}^{s}\lambda_i(\sum_{j=1}^{s} p_{ij} - 1) + \sum_{(k,h)\in J}\mu_{kh}\, p_{kh}] = 0 \tag{6}$$

$$\sum_{t=1}^{T} y_{t-1,i}\, y_{tj} - \sum_{\mathrm{k}=1}^{s}(\sum_{\mathrm{t}=1}^{\mathrm{T}} y_{t-1,i}\, y_{t-1,\mathrm{k}})\, p_{\mathrm{k}j} = \lambda_i + \mu_{\mathrm{ij}} (\mu_{\mathrm{ij}} = 0 \text{ when (i, j)} \notin \text{J}) \tag{7}$$

Equation (7) can be rewritten into

$$\mathbf{Y - ZP} = \boldsymbol{\lambda\xi}^{\mathbf{T}} + \mathbf{U} \tag{8}$$

where $[\mathbf{Y}]_{ij} \equiv \sum_{t=1}^{T} y_{t-1,i}\, y_{tj}$

$[\mathbf{Z}]_{ij} \equiv \sum_{t=1}^{T} y_{t-1,i}\, y_{t-1,j}$

$[\boldsymbol{\lambda}]_i \equiv \lambda_i$

$[\mathbf{U}]_{ij} \equiv \mu_{ij}$

$[\boldsymbol{\xi}]_i \equiv 1$

When all of $p_{kh} \neq 0$, then all of $\mu_{\mathrm{kh}} = 0$, which means $\mathbf{U} = \mathbf{0}$. Therefore,

$$\mathbf{P} = \mathbf{Z}^{-1}\mathbf{Y} \tag{9}$$

In the calculation, Equation (9) was used at first. As is clear from Equation (6), the calculated transition probability matrices automatically satisfy the Equation (5), however, the calculated transition probability does not necessarily take value between 0 and 1. When negative value was obtained, the corresponding transition probability was assumed to be 0. Then the transition probability matrices were calculated again as follows [3].

μ_{ij} was calculated from the following equations.

$$\mathbf{P} = \mathbf{Z}^{-1}(\mathbf{Y} + \frac{\mathbf{1}}{\mathbf{s}}\mathbf{U}\boldsymbol{\xi\xi}^{\mathbf{T}} - \mathbf{U}) \tag{10}$$

$$p_{kh} = 0 \quad (k,h) \in J$$

Then, $\mathbf{P}$ can be calculated by substituting the μ_{ij} into the equation (10).

Using the above method [Method 1], the transition probability matrices were estimated for damages of Type 1 and Type 2. The estimated transition probability matrices are shown in the Table 3. The relative frequency distributions predicted from the transition probability matrices are shown in the Figures 1.1-1.3. for corrosion of steel main girders, spalling/ exposure of reinforcement of concrete deck, and crack of concrete deck.

Since the frequencies other than A, B and C were very few, only the three states were considered in the calculation. The figures at the top show the inspection results. The figures in the middle show the relative frequency distributions predicted from the following equation [Prediction 1].

$$\hat{\boldsymbol{\pi}}(n) = \boldsymbol{\pi}(n-1)\mathbf{P} \qquad (1')$$

where, $\hat{\boldsymbol{\pi}}(n)$: predicted state probability vector at time n.

The figures at the bottom show the relative frequency distributions predicted from the following equation [Prediction 2].

$$\hat{\boldsymbol{\pi}}(n) = \boldsymbol{\pi}(0)\mathbf{P}^{n} \qquad (1'')$$

The prediction errors defined as in Equation (2) are also shown in the figures. The predicted relative frequency distributions agree fairly well with the inspection results.

Discussions

Corrosion of steel main girders and spalling/ exposure of reinforcement of concrete deck belong to the damages of Type 1, where damage rating changes to higher level with age. On the other hand, crack of concrete deck belong to the damages of Type 2, where damage rating does not change to higher level with age. The transition probability p_{AA} of Type 1 is larger than that of Type 2, however, the transition probability p_{BA} of Type 2 is larger than that of Type 1.

Transition probabilities in the lower left of the transition probability matrix, p_{BA} for example, show the effects of repair or rehabilitation. If no repair or rehabilitation work is done, lower left components of the matrix should be equal to 0. The information on the repair or rehabilitation is not available for these data. If it is assumed that repair or rehabilitation were conducted, and that these effects were accurately reflected in these transition probabilities, then transition probability matrix in case of no repair or rehabilitation can be obtained as in the Table 4.2.

TABLE 4.1 ORIGINAL TRANSITION PROBABILITY MATRIX

	A	B	C
A	p_{AA}	p_{AB}	p_{AC}
B	p_{BA}	p_{BB}	p_{BC}
C	p_{CA}	p_{CB}	p_{CC}

TABLE 4.2 MODIFIED TRANSITION PROBABILITY MATRIX
IN CASE OF NO REPAIR OR REHABILITATION

	A	B	C
A	p_{AA}	p_{AB}	p_{AC}
B	0	$p_{BB}/(p_{BB}+p_{BC})$	$p_{BC}/(p_{BB}+p_{BC})$
C	0	0	1

If it is assumed that no repair or rehabilitation was conducted, transition probability matrices can be calculated applying the Equation (4) to the lower left components of the matrix [Method 2]. The result is shown in the Figure 2.1 for the corrosion of steel main girders. p_{AA} or p_{BB} in the matrix of the Figure 2.1 is larger than those in the Figure 1.1 where it was assumed that repair or rehabilitation was conducted. The errors shown in the middle figures are not so different between the Figure 1.1 and the Figure 2.1, however, the error shown in the bottom figure of the Figure 2.1 is much larger than that of Figure 1.1.

In the prediction of the transition probability matrices in the Figure 1.1 and the Figure 2.1, the following prediction errors for the middle figures were minimized.

$$\textit{prediction error}\ \varepsilon 1 \equiv \sum_{t=1}^{T}\sum_{j=1}^{s}(y_{tj} - \sum_{i=1}^{s} y_{t\text{-}1,i}\, p_{ij})^2$$

On the other hand, prediction errors for the bottom figures are as follow.

$$\textit{prediction error}\ \varepsilon 2 \equiv \sum_{t=1}^{T}\sum_{j=1}^{s}(y_{tj} - \sum_{i=1}^{s} \hat{y}_{t\text{-}1,i}\, p_{ij})^2$$

where, $\hat{y}_{0,i} = y_{0,i}$

$$\hat{y}_{t,i} = \sum_{k=1}^{s} \hat{y}_{t\text{-}1,k}\, p_{ki} \qquad t \geq 1$$

By minimizing the error $\varepsilon 2$, the third transition probability matrix was predicted [Method 3], and the results are shown in the Figure 2.2. In the prediction, p_{AC} as well as lower left components were assumed to be 0. Although the error in the middle figure was slightly larger than those in the Figures 1.1 and 2.1, the error in the bottom was much smaller as was expected.

Since the information on the repair or rehabilitation is not available for these data, we cannot conclude the lower left components of the matrix. The problem will be solved by analyzing transition data obtained from two consecutive inspection results of the same member of the same bridge, which was not repaired or rehabilitated between the two inspections.

Conclusions

Bridge members' damage characteristics were studied using the inspection records. Some of the findings are as follows.

1. Damages can be classified into three types. It was found that most of the damages were classified into Type 3, where damage rating other than A was scarcely observed.

2. The transition probability matrices were estimated for all the bridge members' damages except for the Type 3, using the damage rating records of the bridges up to the age of 40 years. The transition probability was determined so that prediction error may be minimum. The relative frequency distributions predicted from the transition probability matrices agreed fairly well with the inspection results. The transition probability p_{AA} of Type 1 was larger than that of Type 2, however, the transition probability p_{BA} of Type 2 was larger than that of Type 1.

3. The transition probability matrices depend on the information on the repair or rehabilitation conducted to the bridge. Since the information was not available for the data we analyzed, we cannot determine the lower left components of the matrix. The problem will be solved by analyzing transition data obtained from two consecutive inspection results of the same member of the same bridge, which was not repaired or rehabilitated between the two inspections.

References

1) National Highway and Risk Management Division, Road Bureau, MLITT: Periodical Inspection Manual for Bridges (Draft), 2004, 3, (in Japanese)
2) H. Sato: Distribution of damage rating of bridge members and a few considerations, 24th US - Japan Bridge Engineering Workshop, 2008, 9
3) H. Morimura and Y. Takahashi: Markov Analysis, JUSE Press, Ltd, 1979, pp.287-291 (in Japanese)

TABLE 2 BRIDGE MEMBERS AND THEIR DAMAGES

Members	**Steel Main Girder**	**Steel Cross Beam**	**Steel Stringer**	**Steel Plate Deck**	**Steel Abutment or Pier**	**Steel Bearing**	**Steel Expansion Joint**
Damages	Corrosion	Corrosion	Corrosion	Corrosion	Corrosion	Corrosion	Corrosion
	Crack	Crack	Crack	Crack	Crack	Crack	Crack
	Looseness or Falling off	Looseness or Falling off	Looseness or Falling off	Looseness or Falling off	Looseness or Falling off	Looseness or Falling off	Looseness or Falling off
	Fracture	Fracture	Fracture	Fracture	Fracture	Fracture	Fracture
	Paint Failure	Paint Failure	Paint Failure	Paint Failure	Paint Failure	Paint Failure	Paint Failure
	Unusual Gap		Unusual Gap	Unusual Gap		Function Failue	Unusual Gap
	Unusual Sound or Vibration	Unusual Sound or Vibration	Unusual Sound or Vibration	Unusual Sound or Vibration	Unusual Sound or Vibration	Water Leakage or Ponding	Surface Roughness
	Unusual Deflection	Unusual Deflection	Unusual Deflection	Unusual Deflection	Unusual Deflection	Deformation or Defect	Deformation or Defect
	Deformation or Defect	Deformation or Defect	Deformation or Defect	Deformation or Defect	Deformation or Defect	Dirt and Debris	Dirt and Debris
						Sag, Move or Slope	
Members	**Concrete Main Girder**	**Concrete Cross Beam**	**Concrete Stringer**	**Concrete Deck**	**Concrete Abutment or Pier**	**Other Bearing**	**Other Expansion Joint**
Damages	Crack	Crack	Crack	Crack	Crack	Fracture	
	Spalling or Exposure of Reignforcement	Spalling or Exposure of Reignforcement	Spalling or Exposure of Reignforcement	Spalling or Exposure of Reignforcement	Spalling or Exposure of Reignforcement		
	Water Leakage or Efflorescence	Water Leakage or Efflorescence	Water Leakage or Efflorescence	Water Leakage or Efflorescence	Water Leakage or Efflorescence		
	Damage of Reignforcement	Damage of Reignforcement	Damage of Reignforcement	Damage of Reignforcement	Damage of Reignforcement		
	Delamination	Delamination	Delamination	Delamination	Delamination		
	Unusual Gap	Unusual Gap	Unusual Gap	Unusual Gap		Function Failue	Unusual Gap
	Anchor Problem	Anchor Problem	Anchor Problem	Anchor Problem	Anchor Problem		Surface Roughness
	Change of Colour	Change of Colour	Change of Colour	Change of Colour	Change of Colour	Change of Colour	Change of Colour
	Water Leakage or Ponding	Water Leakage or Ponding	Water Leakage or Ponding	Water Leakage or Ponding	Water Leakage or Ponding	Water Leakage or Ponding	Water Leakage or Ponding
	Unusual Sound or Vibration	Unusual Sound or Vibration	Unusual Sound or Vibration	Unusual Sound or Vibration	Unusual Sound or Vibration		Unusual Sound or Vibration
	Unusual Deflection	Unusual Deflection	Unusual Deflection	Unusual Deflection	Unusual Deflection		
	Deformation or Defect	Deformation or Defect	Deformation or Defect	Deformation or Defect	Deformation or Defect	Deformation or Defect	Deformation or Defect
				Falling off		Dirt and Debris	Dirt and Debris

Legend

(yellow)	: Transition Probability Matrices were predicted.
others	: Transition Probability Matrices were not predicted because damage ratings other than A were scarcely observed.

TABLE 3 TRANSITION PROBABILITY MATRICES OF BRIDGE MEMBERS' DAMAGE RATING

Steel Main Girder, Corrosion			Steel Cross Beam, Corrosion			Steel Stringer, Corrosion			Steel Plate Deck, Corrosion		
0.925	0.075	0.000	0.955	0.038	0.007	0.904	0.078	0.018	0.858	0.115	0.027
0.220	0.553	0.227	0.265	0.541	0.195	0.465	0.372	0.163	0.242	0.542	0.216
0.000	0.949	0.051	0.000	1.000	0.000	0.251	0.612	0.137	0.071	0.801	0.128

Steel Abutment or Pier, Corrosion			Steel Bearing, Corrosion			Steel Expansion Joint, Corrosion		
0.836	0.116	0.047	0.899	0.101	0.000	0.904	0.096	0.000
0.520	0.384	0.096	0.226	0.542	0.231	0.505	0.420	0.076
0.434	0.463	0.103	0.000	0.660	0.340	0.000	0.807	0.193

Steel Main Girder, Paint Failure			Steel Cross Beam, Paint Failure			Steel Stringer, Paint Failure			Steel Plate Deck, Paint Failure		
0.707	0.277	0.017	0.767	0.204	0.029	0.588	0.356	0.056	0.833	0.158	0.009
0.323	0.602	0.075	0.400	0.520	0.080	0.672	0.239	0.089	0.236	0.647	0.118
0.000	0.824	0.176	0.191	0.602	0.208	0.146	0.514	0.340	0.000	0.811	0.189

Steel Abutment or Pier, Paint Failure			Steel Bearing, Paint Failure			Steel Expansion Joint, Paint Failure		
0.646	0.322	0.032	0.791	0.209	0.000	0.936	0.064	0.000
0.655	0.271	0.074	0.339	0.504	0.157	0.504	0.445	0.050
0.632	0.253	0.115	0.000	0.932	0.068	0.000	1.000	0.000

Concrete Main Girder, Crack			Concrete Cross Beam, Crack			Concrete Deck, Crack			Concrete Abutment or Pier, Crack		
0.966	0.031	0.004	0.920	0.077	0.003	0.658	0.337	0.005	0.604	0.384	0.012
0.569	0.431	0.000	0.694	0.306	0.000	0.439	0.542	0.019	0.864	0.136	0.000
1.000	0.000	0.000	1.000	0.000	0.000	0.021	0.444	0.535	1.000	0.000	0.000

Concrete Main Girder, Spalling or Exposure of Reignforcement			Concrete Cross Beam, Spalling or Exposure of Reignforcement			Concrete Deck, Spalling or Exposure of Reignforcement			Concrete Abutment or Pier, Spalling or Exposure of Reignforcement		
0.973	0.026	0.002	0.971	0.029	0.000	0.981	0.013	0.006	0.991	0.009	0.000
0.615	0.251	0.134	0.330	0.535	0.134	0.000	0.912	0.088	0.053	0.808	0.139
0.000	0.941	0.059	0.000	0.611	0.389	0.030	0.295	0.675	0.000	0.986	0.014

Concrete Cross Beam, Water Leakage or Efflorescence			Concrete Deck, Water Leakage or Efflorescence			Concrete Abutment or Pier, Water Leakage or Efflorescence		
0.819	0.175	0.006	0.693	0.307	0.000	0.957	0.043	0.000
0.817	0.183	0.000	0.216	0.738	0.046	0.263	0.712	0.025
0.704	0.228	0.067	0.153	0.537	0.309	0.000	1.000	0.000

Concrete Deck, Delamination		
0.991	0.009	0.000
0.105	0.648	0.246
0.000	0.297	0.703

Other Expansion Joint, Water Leakage or Ponding			Concrete Abutment or Pier, Water Leakage or Ponding		
0.846	0.094	0.061	0.954	0.045	0.001
1.000	0.000	0.000	0.420	0.549	0.030
0.966	0.034	0.000	0.481	0.376	0.143

Other Bearing, Change of Colour			Other Expansion Joint, Change of Colour		
0.978	0.022	0.000	0.946	0.038	0.016
0.438	0.545	0.017	0.571	0.427	0.002
1.000	0.000	0.000	0.070	0.569	0.361

Other Expansion Joint, Deformation or Defect		
0.965	0.025	0.010
0.530	0.358	0.111
0.575	0.329	0.097

Legend:

Member, Damage		
P_{AA}	P_{AB}	P_{AC}
P_{BA}	P_{BB}	P_{BC}
P_{CA}	P_{CB}	P_{CC}

A:No damage, or the damage is so light that repair is unnecessary.
B:Repair is necessary according to the situations.
C:Prompt repair or other work is necessary.

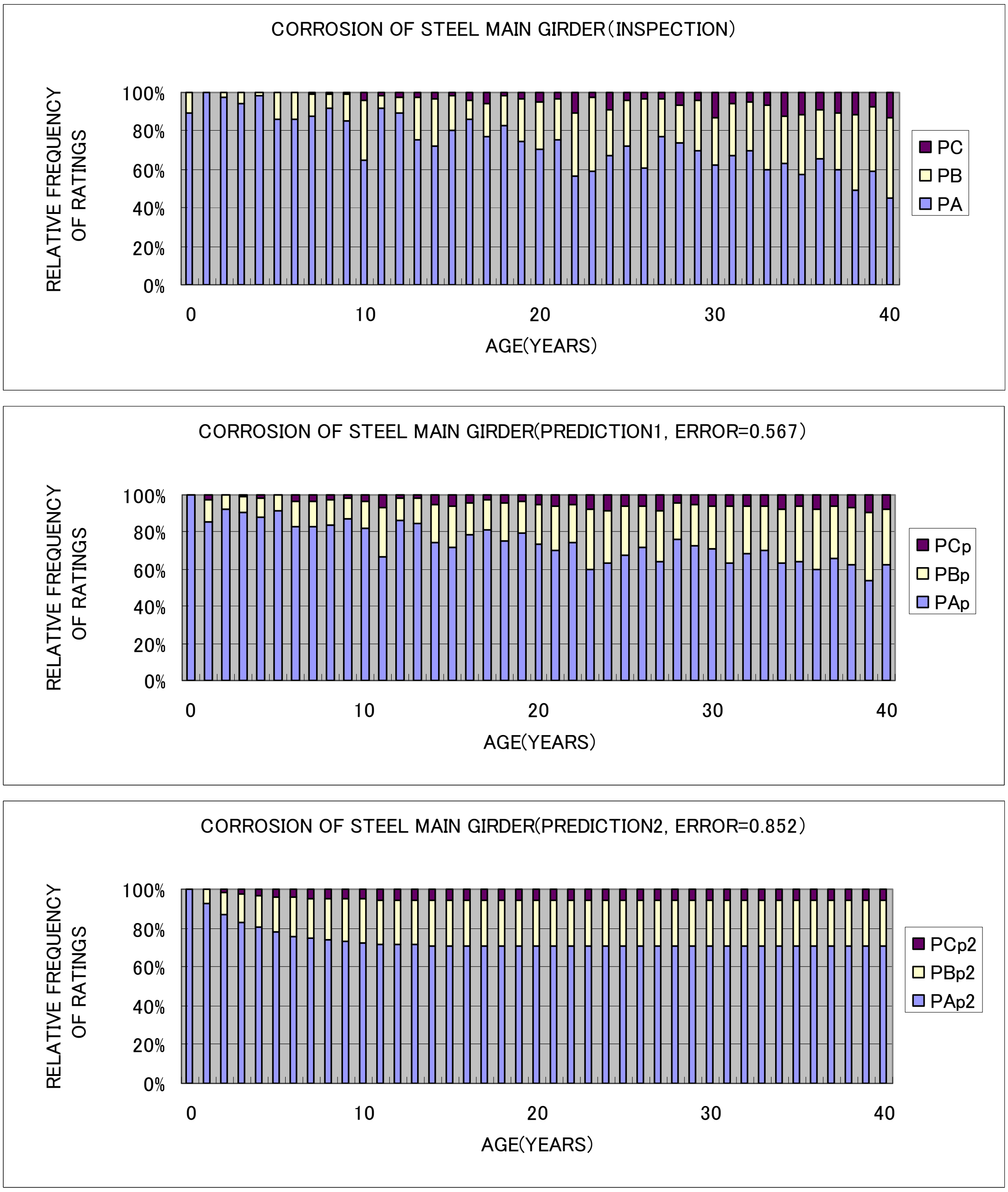
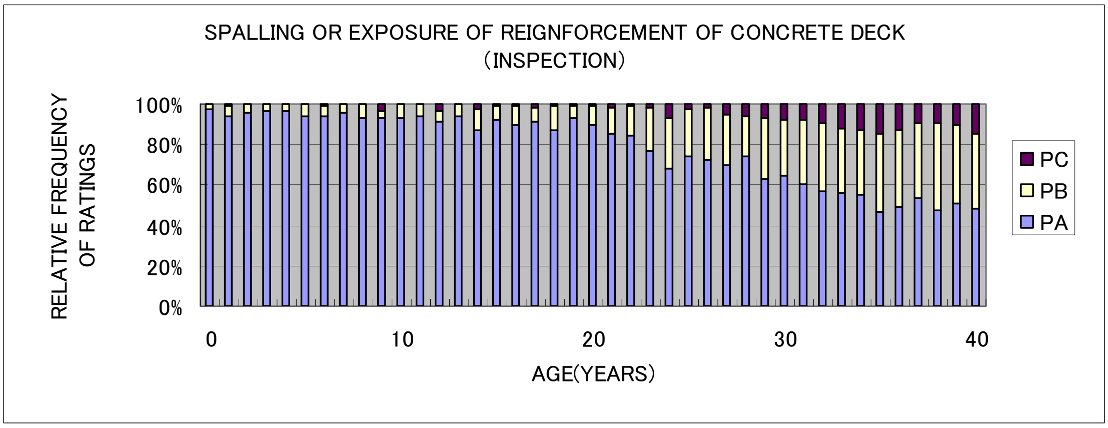
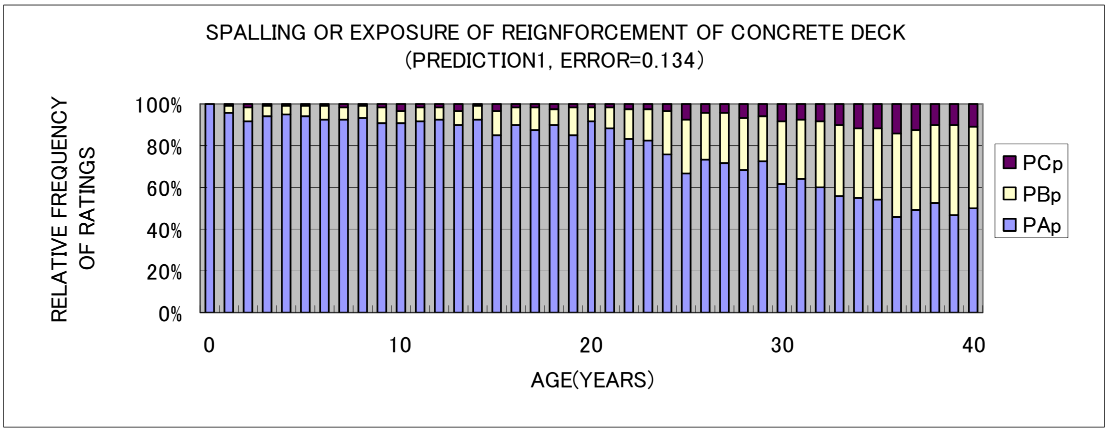


FIG. 1.1 DISTRIBUTION OF DAMAGE RATING FOR CORROSION OF STEEL MAIN GIRDERS (TOP: INSPECTION (SOURCE: MINISTRY OF LAND, INFRASTRUCTURE, TRANSPORT, AND TOURISM), MIDDLE: PREDICTION1, BOTTOM: PREDICTION2, RIGHT: TRANSITION PROBABILITY MATRIX OBTAINED FROM METHOD1)

	A	B	C
A	0.925	0.075	0.000
B	0.220	0.553	0.227
C	0.000	0.949	0.051





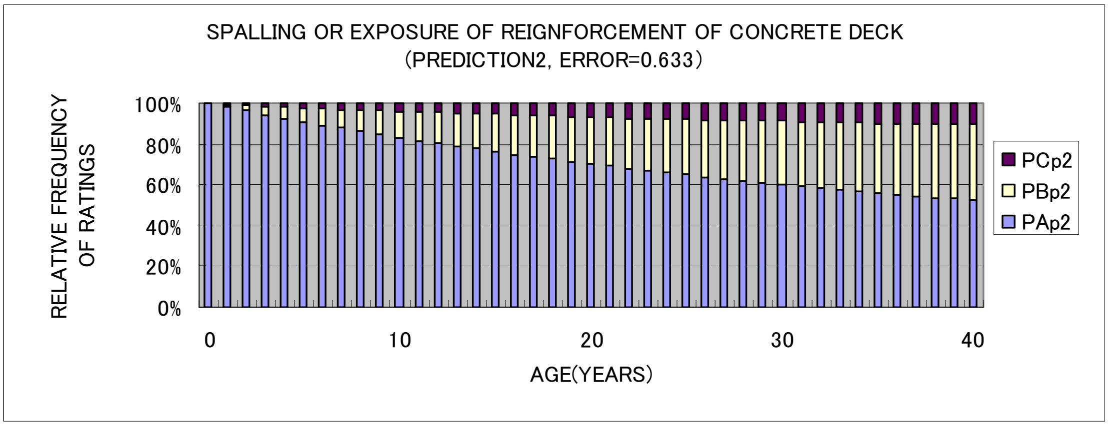


FIG. 1.2 DISTRIBUTION OF DAMAGE RATING FOR SPALLING OR EXPOSURE OF REIGNFORCEMENT OF CONCRETE DECK (TOP: INSPECTION (SOURCE: MINISTRY OF LAND, INFRASTRUCTURE, TRANSPORT, AND TOURISM), MIDDLE: PREDICTION1, BOTTOM: PREDICTION2, RIGHT: TRANSITION PROBABILITY MATRIX OBTAINED FROM METHOD1)

	A	B	C
A	0.981	0.013	0.006
B	0.000	0.912	0.088
C	0.030	0.295	0.675

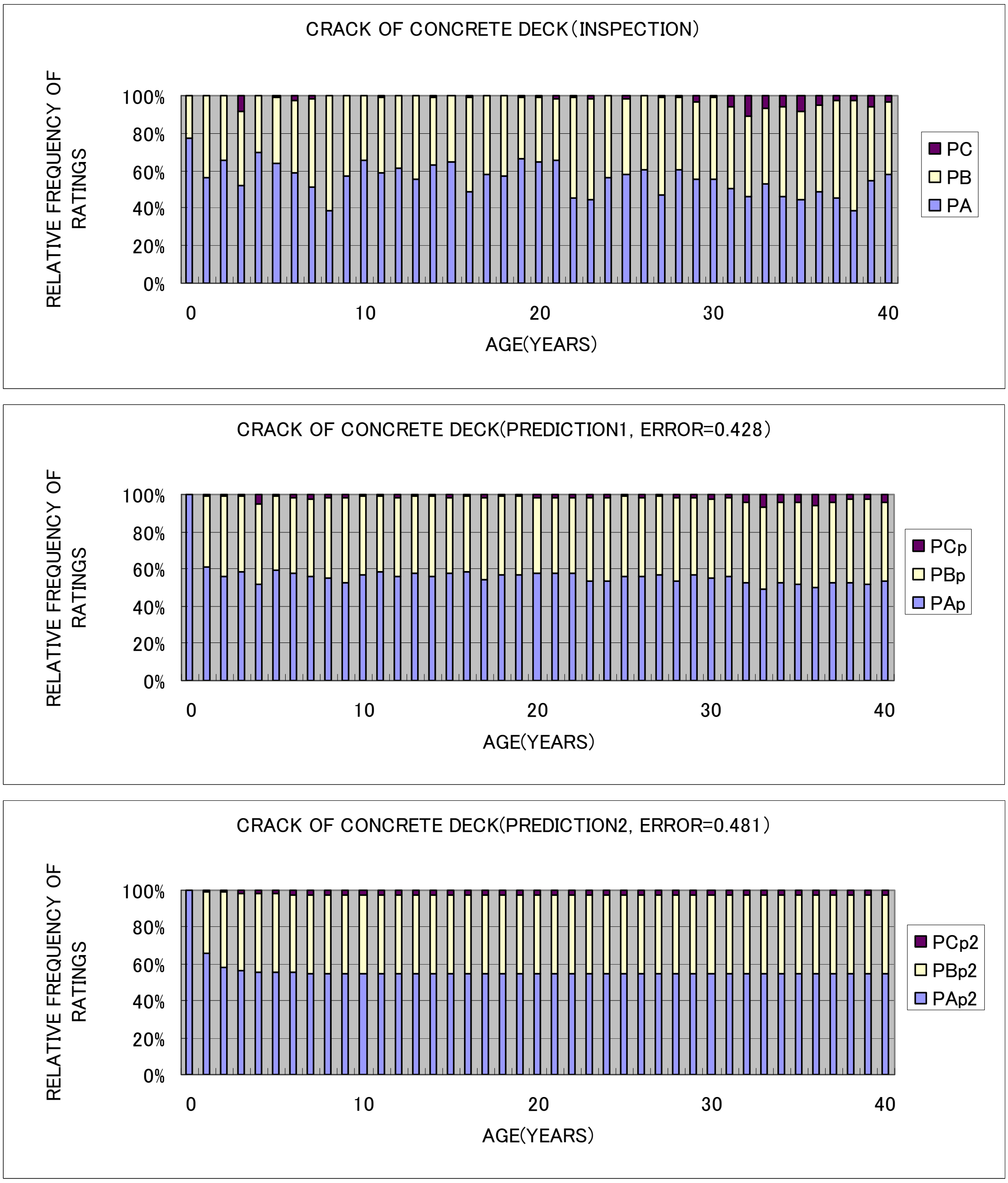


FIG. 1.3 DISTRIBUTION OF DAMAGE RATING FOR CRACK OF CONCRETE DECK (TOP: INSPECTION (SOURCE: MINISTRY OF LAND, INFRASTRUCTURE, TRANSPORT, AND TOURISM), MIDDLE: PREDICTION1, BOTTOM: PREDICTION2, RIGHT: TRANSITION PROBABILITY MATRIX OBTAINED FROM METHOD1)

	A	B	C
A	0.658	0.337	0.005
B	0.439	0.542	0.019
C	0.021	0.444	0.535

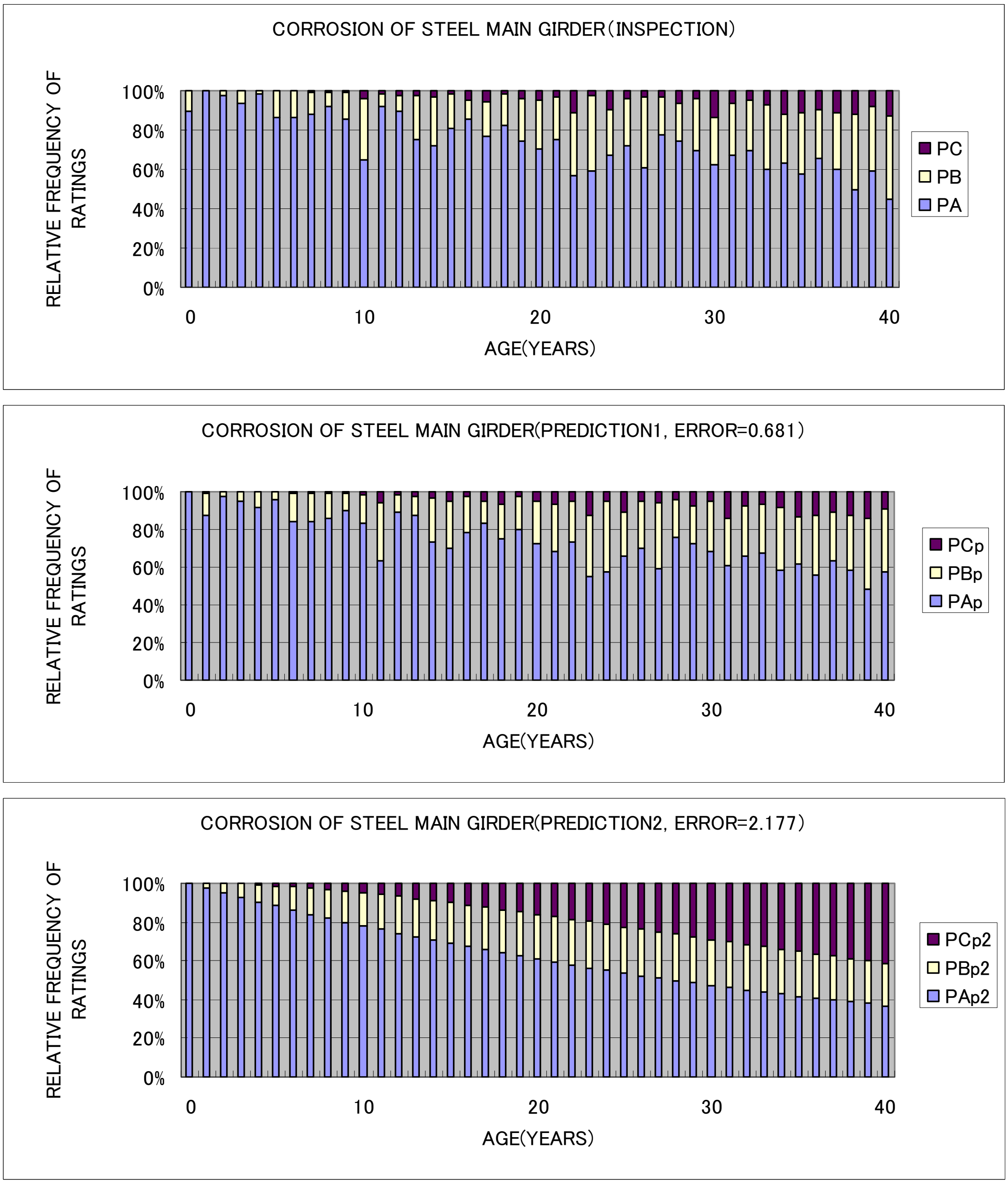


FIG. 2.1 DISTRIBUTION OF DAMAGE RATING FOR CORROSION OF STEEL MAIN GIRDERS (TOP: INSPECTION (SOURCE: MINISTRY OF LAND, INFRASTRUCTURE, TRANSPORT, AND TOURISM), MIDDLE: PREDICTION1, BOTTOM: PREDICTION2, RIGHT: TRANSITION PROBABILITY MATRIX OBTAINED FROM METHOD2)

	A	B	C
A	0.975	0.025	0.000
B	0.000	0.946	0.054
C	0.000	0.000	1.000

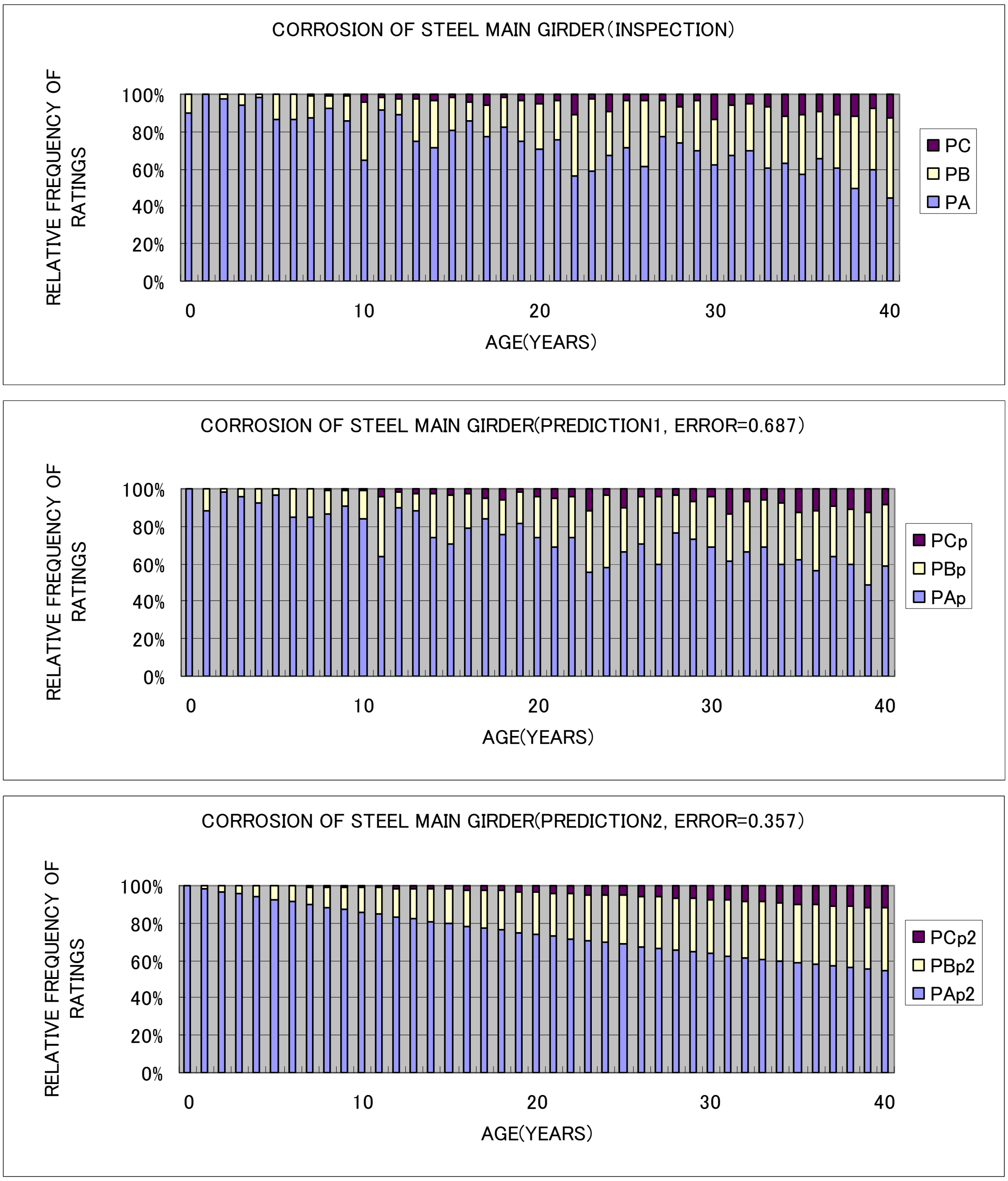


FIG. 2.2 DISTRIBUTION OF DAMAGE RATING FOR CORROSION OF STEEL MAIN GIRDERS (TOP: INSPECTION (SOURCE: MINISTRY OF LAND, INFRASTRUCTURE, TRANSPORT, AND TOURISM), MIDDLE: PREDICTION1, BOTTOM: PREDICTION2, RIGHT: TRANSITION PROBABILITY MATRIX OBTAINED FROM METHOD3)

	A	B	C
A	0.985	0.015	0.000
B	0.000	0.985	0.015
C	0.000	0.000	1.000