PERFORMANCE-BASED SEISMIC DESIGN AND ASSESSMENT OF BRIDGES

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Abstract

In modern seismic-resistant bridge design, there is a need for bridge designs and technologies that are both sustainable and resilient; however, quantifying both of these is challenging. Performance-based earthquake engineering as applied to typical highway bridges is presented in this paper with the emphasis on 1.) defining performance using damage, loss, and sustainability metrics, 2.) probabilistic quantification of performance, and 3.) differentiation between performance-based assessment and design. Examples are presented for performance-based design using a case study purposely kept simple, followed by performance-based assessment of an integrated bridge-ground system. Rapid assessment of these complex models is enabled through recently developed software.

Introduction

Traditional approaches to seismic design of infrastructure components, such as bridges, involve use of prevailing codes, standards, and guidelines pertinent to the materials employed, the hazard, and geographic region. These approaches are largely prescriptive in nature and have the benefit of being easy to execute in a sequential manner, or occasionally with minimal iteration. These prescriptive approaches should not be interpreted as entirely without assessment, as there are often analytical steps taken after preliminary design to confirm certain performance criteria or limits are satisfied. However, the assessment is rarely risk based and is not provided in metrics suitable for owners and stakeholders.

The problem of performance-based earthquake engineering (PBEE) is better understood in the reverse direction: the desired or target level of performance is specified apriori and the design is executed (almost certainly in an iterative fashion) to ensure this level of performance is met. Design of infrastructure using a PBEE approach, commonly abbreviated as performance-based seismic design (PBSD) or performance-based design (PBD), is not yet a reality. There is currently no basis (research or practitioner) for taking target performance and translating it into the design of an infrastructure component, let alone the detailing of individual members or constituent materials within the infrastructure component. While commonly labeled as PBD, the majority of the body of knowledge that has been accumulated to date on PBEE is actually on the single forward problem of performance-based seismic assessment (PBSA).

In PBSA, a single known realization or bridge design is assessed to better
understand the seismic performance. Typically this is executed in the as-built condition and does not necessarily include maintenance, deterioration, or multiple occurrences of a given hazard (whether it’s live load over time or seismic load in this case). By evaluating several design schemes or realizations in parallel (i.e., perform several different PBSAs), one is able to compare the response of different designs and has therefore sometimes been called PBSD in the past. But a central concept to both PBSA and PBSD relates to the definition of performance. PBEE aims to quantify the seismic performance and risk of engineered facilities using metrics that are compatible with engineers, owners, stakeholders, and managers alike. Therefore, often the definition of performance follows the function of the infrastructure components.

For example, bridges may have damage and downtime associated with the structure itself, but the consequences of interest to society are related to the performance of the transportation network as a whole. Therefore, metrics utilized in network risk assessment would be appropriate when assessing highway bridges. The complication in these varied definitions of performance is that PBEE is more general than the typical purview of the structural, geotechnical, or construction engineer. Evaluation of performance metrics that include consequences requires not only quantification of seismic hazard, structural response, and resulting damage, but also the relationship between that damage and the ultimate performance sought. This mapping between response during the earthquake, damage, and consequences is often termed loss modeling.

The objective of this paper is to demonstrate both PBSA and PBSD of typical concrete highway bridges specifically considering potential losses as the measures of performance. PBSD is illustrated using a model purposely kept simple to facilitate analysis, followed by PBSA of a typical highway overpass using a more complex integrated bridge-foundation-ground model. The performance metrics presented are repair costs and repair times. In addition, software developed to facilitate such PBSAs is introduced, known as BridgePBEE, with some results showing how interesting studies can be performed using such a tool. Specifically, loss hazard maps are generated for the California based on the loss metrics developed above. Finally, the paper introduces the next generation of performance metrics that relate to environmental consequences. Sustainability metrics are introduced into the PBSA framework, as measured in terms of carbon footprint, although full bridge analyses are still being researched and developed.

**Performance-based Framework**

By definition, PBEE requires treatment of the underlying uncertainties inherent in defining both the hazard and the properties of the bridge and surrounding site. Fundamentally, the PBEE problem can only be defined in terms of probabilities, confidence levels, or risk, because neither the hazard nor the properties are deterministic in nature. The concept of probability in seismic design is not new, the language is already incorporated directly into existing codes. For example, FEMA 350 gives the example of
“a design may be determined to provide a 95% level of confidence that the structure will provide Collapse Prevention or better performance for earthquake hazards with a 2% probability of exceedance in 50 years”. This is typical of the PBEE language where the performance objective is stated as a combination of performance level that must be achieved for the infrastructure component and a hazard level, both described probabilistically.

The PBEE framework utilized in this paper includes several building blocks (intermediate probabilistic models) that allow better quantification of the relationship between the hazard, response, damage, and consequences. A hazard model uses earthquake ground motion data to define hazard and recorded time histories to use for simulation. The quantity used to categorize the intensity of each earthquake is referred to as an intensity measure (IM). The demand model uses response from dynamic analysis to determine an engineering demand parameter (EDP) and the relationship to earthquake IM. Demand models are traditionally obtained using nonlinear time history analysis. The damage model connects the EDP to a damage measure (DM) or discrete set of damage states (DS). The damage model is commonly obtained from available experimental data sets or expert opinion. Finally the loss model characterizes the consequences due to the states of damage, and is usually presented as a decision variable (DV).

The simplest way to characterize DVs is to generate a series of probabilistic models as described above that directly relate each successive quantity to the previous one. The consequence is only a single, or scalar, variable in each model. This level of simplicity is used to illustrate a PBSD approach in this paper, and is also appropriate for assessing larger numbers of bridges such as for network risk assessment. However, it is also commonly recognized that the damage to bridges (and other structures) may be localized and certainly the consequences of damage in different elements or subassemblies are not equal. Therefore, many researchers have proposed vector relationships for each of the probabilistic models. In this paper, such an assembly-based procedure is adopted to show the PBSA of a single bridge-ground system.

In the assembly-based approach, the bridge is divided into performance groups (PGs). PGs represent a collection of structural components that act as a global-level indicator of structural performance and that contribute significantly to repair-level decisions. PGs are not necessarily the same as load-resisting structural components; however, they do imply that the demand model establish the response (EDP) for each PG using a different variable. Each PG then is described using a unique DM or discrete set of DSs. The procedure deviates from the simple approach mentioned previously, because of the introduction of a repair model between damage and loss models. The repair model describes repair methods and repair quantities (Q) necessary to return the DSs to original functionality. Finally, the Qs are reassembled between all PGs (hence the term assembly-based) to form realistic consequences in the loss model. The examples presented in this paper consider only repair cost and repair time as the DVs, but the paper also introduces sustainability as a performance metric in the last section.
**Performance-based Seismic Design (PBSD)**

Recent efforts in the earthquake engineering community have focused largely on developing a PBSA methodology. However, equally important is applying such a methodology to the design of structures. Probabilistic PBSD enables design and acceptance criteria to be specified by a continuum of limit states and incorporate uncertainty in all the sources of data. Here two brief examples are presented that show the design of simple bridge structures in terms of single or multiple physical design parameters, such as column height and diameter. The bridge structures considered come from a family of parameterized reinforced concrete California highway box girder overpass bridges utilized in earlier studies by Mackie and Stojadinovic (2005). An elevation view of the (3D) bridge models with design parameters indicated is shown in FIGURE 1. As with all the examples presented in this paper, the IM selected is peak ground velocity (PGV).

![Figure 1 - Elevation view of parameterized bridge models used for PBSD](image)

Design parameter solutions are obtained from explicit consideration of aleatory and epistemic uncertainty in the hazard, demand, damage, and loss to the structure by way of specifying a level of confidence in the resulting design under different performance objectives. The performance levels considered were either damage (focusing on column damage) or loss (defined in terms of repair costs). As mentioned previously, the probabilistic models were purposely kept simple to facilitate the design parameter solution. Specifically, it was assumed that the damage and loss models were scalar (as opposed to the vector or assembly-based approach demonstrated for PBSA in the next section). The subsequent design approaches are denoted performance-based damage design (PBDD) for damage performance criteria and performance-based loss design (PBLD) for decision (or loss) criteria. Complete documentation of the derivation and examples can be found in Mackie and Stojadinovic (2007).
While based on many assumptions, the design methodology is unique because of the continuum of damage and loss performance levels, the design procedure does not require numerous performance assessments to iteratively converge on a design solution, and the solution of the design equations provide physical design parameters of interest to bridge designers (such as column diameters and heights). Closed-form expressions were derived in Mackie and Stojadinovic (2007) for both a performance-checking criterion (for the damage and loss performance levels) as well as for design. Both types of expressions depend on the ability to derive the mean annual frequency (MAF) of exceeding either damage or loss performance levels. The MAF is commonly understood from probabilistic seismic hazard analysis (PSHA) and, under some assumptions, can be extended to the other intermediate variables to generate structure-specific hazard curves measured by EDPs, DMs, or DVs.

The first PBSD example considers the case where only a single design parameter is unknown. The span length-to-column height ratio (L/H) is used as the design parameter. The span length is held fixed for the bridge while varying the design parameter; therefore, the bridge column height is increased directly as the design parameter is decreased. While the design procedure is non-iterative, it does require that the performance of bridges within the sample-space of parameters has been previously defined (i.e., several PBSAs have already been performed). The performance-checking criterion is analogous to finding the level of confidence associated with select factored-demand to factored-capacity ratios. For example, the confidence level of not realizing a loss state of 25% repair cost ratio (confidence of repair cost being less than 25% of replacement cost) is shown in FIGURE 2 for two possible bridge designs (L/H values). For a fixed IM, the confidence of being under the 25% RCR increases with L/H (i.e., taller column).

If the design parameter is not known, design curves for fixed confidence levels can be found. Such a set of design curves is presented with the continuum of hazard levels in FIGURE 3. This shows how the column height needs to be changed to effect the desired performance, here measured in terms of a RCR. However, in a real world design situation, it is unlikely that all but a single design parameter will be constrained by functional or economic objectives. Therefore, it is instructive to derive solutions for multiple design parameters for a single confidence level and target performance realization. The multiple design parameter application is considerably more complex as there may exist infinite solutions for the vector of design parameters $\mathbf{y}$ in the PBLD design equation. Therefore, a constrained nonlinear solution algorithm was used to solve for the vector solution of design parameters ($\mathbf{y}^*$) in the interval of data from which the original design equations were derived. The solution closest to a point specified by the designer ($\mathbf{y}_0$) that best describes the requirements of the bridge site at hand was used in Mackie and Stojadinovic (2007).
A vector of two design parameters is illustrated here using $y = [L/H, D_c/D_s]$, where $D_c/D_s$ is the ratio of column diameter to superstructure depth. Two design parameter solutions are easily plotted in three-dimensional space and the functions governing the demand are less complex; however, it does not preclude the method being used for larger design parameter spaces. It was assumed that an initial design parameter estimate of $y_0 = [2.4, 0.75]$ was appropriate for the bridge under consideration. Following the same logic as before, performance-checking criteria for damage and loss states can be derived with known realizations of the design parameters. However, more interesting is the optimal solution of the design equations to obtain target confidence levels of performance defined by the RCR being less than 25%. These 3D design curves are shown in FIGURE 4. The vertical line marked with crosses shows the location of the $y_0$ values.
Performance-based Seismic Assessment (PBSA)

PBEE and PBSA, as applied to buildings, have seen rapid development and adoption recently (e.g., ATC-58 and ATC-63). However, in the bridge and infrastructure arena, there have been fewer attempts (e.g., Mackie et al. 2008; Solberg et al., 2008; Bradley et al., 2010) at rigorous development of the data necessary for PBEE or packaging the tools in a form that allows rapid PBEE-based evaluation and assessment such as PACT in ATC-58. Nonlinear analysis under strong ground motion and consideration of coupled soil-foundation-structure effects may not be warranted for the design of individual highway overpass bridges. However, assessment of an inventory of bridges in a network is unreliable using generic class fragilities due the variability in structural configurations, site conditions, and hazard. Therefore, the development of enabling technologies that allow PBSA is required, because the problem is not tractable if each scenario requires detailed knowledge in ground motion hazard, structural modeling, soil modeling, component and system-level damage assessment, and post-earthquake consequences.

BridgePBEE (http://peer.berkeley.edu/bridgepbee/) was developed as a graphical environment and enabling PBEE technology to address this need. BridgePBEE couples a nonlinear dynamic analysis engine for bridge-ground systems with a PBEE framework. This integration of hazard, modeling, analysis, and damage quantification allows seamless generation of probabilistic repair models conditioned on earthquake intensity for different bridge-ground scenarios, such as those presented in this paper. The performance metrics (or DVs) computed by BridgePBEE are repair costs and repair times. The complete analysis is accomplished using the local linearization repair cost and time methodology (LLRCAT), as detailed in Mackie et al. (2010). The methodology is very efficient at evaluating both probabilities (conditional on IM) of DVs, as well as MAFs of DVs. In addition, the methodology supports the data structures previously developed by Mackie et al. (2008) that allow contributions from each PG to vary at each discrete DS according to input data parameterized on bridge properties (such as geometry, materials, configuration, etc.). The data flows and framework are extensible to other bridge classes; however, to date only repair, cost, and schedule data have been obtained for the typical California reinforced concrete box girder overpass bridges used in the pilot studies and shown in FIGURE 1.

The user interface, soil models, and pile-ground studies that make up the balance of BridgePBEE were part of previous efforts by Elgamal and co-workers (Lu et al., 2006; Elgamal and Lu, 2009). The three-dimensional (3D) ground-foundation graphical user interface OpenSeesPL was the basis for BridgePBEE. This interface allowed for the execution of pushover and seismic single-pile or pile-group ground simulations, and the corresponding pre- and post-processing capabilities. The menu of soil materials in OpenSeesPL included a complementary set of soil modeling parameters representing loose, medium and dense cohesionless soils (with silt, sand or gravel permeability), and soft, medium and stiff clay (J2 plasticity cyclic model). The additions that constituted
BridgePBEE integrated i) a user interface to the PBEE framework, ii) a module for handling the needed input ground motion ensemble and to compute all salient characteristics, iii) modify the graphical interface to automatically generate user-defined bridge-ground FE models, and iv) build the post-processing capability to display the seismic response ensembles, and to display the PBEE outcomes. A sample mesh generated for the analysis performed in this paper is shown in FIGURE 5.

FIGURE 5 - Sample mesh generated within BridgePBEE interface

BridgePBEE was used to conduct PBSA studies on a single bridge configuration (superstructure geometry and materials) founded on four different sites with varying stiffness and strength profiles, ranging from a rigid rock case to a weak upper soil strata case, as would be typical of a network risk assessment (Mackie et al., 2012a). Case 1 is founded on stiff (approaching rigid) soil that is intended to produce a fixed-base structure scenario. The same soil domain geometry (depth, boundary conditions, and extents) is used for the subsequent three cases, which are delineated by different material properties and layer definitions. A sample soil profile used for cast-in-drilled-hole piles (CIDH) and pipe piles was used as a template for the stiffness and strength properties by soil strata that define Case 2 (baseline case). From this typical profile, two variations were derived to highlight the potential influence of the ground layers in terms of base earthquake motion amplification, relative flexibility of the soil around the pile, and susceptibility to the accumulation of permanent deformations. Case 3 has shallow layers with reduced stiffness and strength to highlight the influence of potential permanent deformation. Case 4 is the same as Case 2 but with slightly stiffer upper soil layers to further highlight the role of site amplification.

From the four case studies, complete PBEE computations were performed to obtain intensity-dependent repair cost ratios (RCRs) and repair times (RTs). The mean RCR loss model for each of the four cases is shown in FIGURE 6. The weak soil case
results in the largest RCR for intensities between 20 and 60 cm/s, followed in order of increasing ground stiffness by Case 2, Case 4, and Case 1. However, numerous other interesting conclusions can be drawn as the intensities increase. For example, the rigid base case (Case 1) accumulates the least cost (as would be expected) in addition to reaching a plateau above which the cost does not continue to increase. This is representative of the fact that the foundation PGs do not contribute to the repair and the intensities are not sufficient to cause failure and complete column replacement. Similar behavior is true for both Cases 2 and 4; however, for Case 4 (the stiffest of the three soil scenarios), the costs are actually larger than Case 2. This is due to the reduced distance between peak moments in the pile below grade for Case 4 (due to the stiffer upper layers) that triggers pile repairs at lower residual column pile cap displacements. Finally, the weak soil case also exhibits a cost plateau, but continues to increase rapidly with intensity beyond this due to the accumulation of damage to the foundations that result in RCRs that approach the replacement cost of the bridge.

![FIGURE 6 - Mean RCR loss models for four bridge-ground scenarios](image1)

![FIGURE 7 - MAF of RCR for four bridge-ground scenarios](image2)

From the RCR loss model alone, it is not clear what the contributing bridge PGs or repair quantities are at a given intensity. Therefore, it is often helpful to disaggregate the RCR or RT by PG at each intensity to demonstrate what PG are the primary drivers for repair cost. This data is shown for the two largest cost driver PGs in FIGURE 8. The data shows the delayed onset of repairs to the right abutment (PG4) as the soil profiles are stiffened. The bearing displacements (PG6) are less sensitive to the changes in the soil, except for the rigid ground case where the abutments and column bases are fixed (only superstructure deformations cause abutment and bearing movement). While the PGs shown in the figure result in the highest cost contribution to the bridge RCR, they illustrate how all scenarios considered reach a plateau beyond which costs do not increase (the complete replacement of the component or subassembly). Other ways of visualizing the data are also useful, for example, the expected RCR can also be disaggregated by Q. The breakdown of expected RCR by Q for the weak soil case (Case 3) at different intensities is shown in a pie chart (FIGURE 9).
With the inclusion of ground motion hazard, the loss hazard curve can also be obtained, as shown in FIGURE 7 for the four scenarios. It shows the mean annual frequency (MAF) of exceeding the RCR values on the horizontal axis. The same IM hazard was used for all four scenarios (consistent with the input PGV being used as the IM for the loss models) and provides a clear illustration of the increased cost hazard associated with the decreasing strength and stiffness of the soil profiles. For example, for a fixed hazard level of $5 \times 10^{-4}$, the RCR changes from 2% to 20% between the rigid base and weak soil cases. Similarly, the hazard increases by an order of magnitude at a threshold RCR of 25%. In the hazard curve, the effect of the increased cost of Case 4 over Case 2 at high intensities is diminished due the small rate of occurrence of earthquakes with these large magnitudes.

A follow-on study generated RCR and RT contour maps (Mackie et al., 2012b). The contour maps are similar in format to the ground motion hazard maps commonly
employed in design, but contain additional structure-specific hazard information and allow rapid assessment of spatially-varying systems during a major event. In addition, such loss hazard maps can be used for design screening purposes. They effectively allow a single bridge-ground realization to be “moved” virtually around a stakeholder region (California in this case), and the subsequent effect of the local ground motion hazard on the vulnerability of the structure is reflected in the loss hazard. Results show that different soil conditions can have a large impact on the performance of simple overpass bridges when evaluated in terms of repair cost (instead of only permanent deformations, for example), and that the results can be readily extended to entire regions defined in terms of existing ground motion hazard data.

To be consistent with the PBEE results (and previous probabilistic seismic demand models generated for the bridges), the hazard was converted from PGA to PGV using a conversion factor of 36 in/sec/g. Ideally, the hazard for PGV would be obtained directly or converted from PGA using site-specific ground profile information. However, for comparison purposes in this paper, the same factor was used for all locations and both of the soil scenarios investigated (assuming the ground motions were base input motions). The RCR loss models derived previously were convolved with the ground motion hazard curves, and the procedure was repeated for a multitude of grid points within the state of California to generate the RCR loss hazard maps. Loss hazard maps for a single hazard level (2% probability of exceedance in 75 years) are presented for Case 2 (stiff soil) in FIGURE 10 and Case 3 (weak soil) in FIGURE 11. The 75-year return period was selected to be consistent with hazard characterization in AASHTO. The vertical and horizontal axes show latitude and longitude in units of degrees north and
west, respectively (actually the negative of the longitude is shown).

The 2%-in-75-year probability of exceedance contours peak at 30% RCR and 70% RCR for Case 2 and Case 3, respectively. Clearly, there is a relationship between regions delineated with high loss hazard curves and those with high ground motion hazard contours; however, the loss hazard curves are significantly richer in information content. The intensity-dependent loss models generated by BridgePBEE reflect different repair methods and actions as the ground motion intensity varies. In addition, the rate of accumulation of repair quantities is dependent on the intensity. Therefore, in regions dominated by close distance, high magnitude, events, the RCR distribution will be markedly different than regions where hazard has more uniform contributions from different magnitudes and distances, for example. The ranges of intensity where repair costs do not continue to increase until a more serious repair action is necessary lead to a smoothing of ground motion hazard contours.

**Sustainability Considerations**

Quantifying performance in terms of repair costs or repair times is a useful generalization to stakeholders in assessing, and ultimately designing, structures. However, it remains limited to the domain of life cycle cost analysis (LCCA). True life cycle assessment (LCA) studies; however, should consider the costs to society and the environment in addition to the direct and indirect costs as a consequence of damage from extreme events. Fueled by concerns over global climate change, there is now an interest in quantifying emissions of greenhouse gases (GHGs). Typically, the impact of infrastructure on such emissions is captured using the so-called carbon footprint or embodied energy. The carbon footprint includes all emissions of carbon dioxide (CO₂), or GHG expressed in terms of CO₂ equivalents, that are directly and indirectly caused by an activity (Wiedmann and Minx, 2008).

Carbon footprint analysis is typically performed using process-based LCA, economic input-output (EIO) LCA, or a hybrid version of the two. LCA of processes considers detailed information about all aspects of the construction, operation, maintenance, and demolition of the infrastructure process, but may not include the consequences of the industries used to supply materials to the construction, for example. The converse would be to consider behavior of different sectors (EIO LCA) of the economy rather than details about the process itself. The emissions are estimated across the entire supply-chain in this method and may not underestimate the carbon footprint, as has been the case in some process-based analyses (Wiedmann and Minx, 2008). Carbon footprint calculations are often divided into three scopes. Scope 1 refers to direct emissions from on-site activities. Scopes 2 and 3 account for the indirect emissions that occur off-site. Scope 2 represents emissions from purchased electricity (emissions at generation facility), whereas Scope 3 refers to upstream GHG emissions such from suppliers, production of purchased products, and fuel transportation, amongst other things (WRI and WBCSD, 2004).
Ongoing work on quantifying the carbon footprint of typical highway overpass bridges estimates carbon emissions for the entire supply chain using hybrid LCA analysis. EIO analysis results in multipliers (g CO₂ equivalents per unit) on each material quantity. In fact, the formulation of scope multipliers is related to the estimation of the economic costs (i.e., unit cost) of manufacturing the material. In addition, emissions due to transportation of materials and emissions due to vehicles utilized in the construction process are accounted for in the hybrid LCA. The LCA results in the same multipliers regardless of whether the process is viewed as initial construction, maintenance, or repair after an extreme event. However, the construction methods change for these discrete events and are reflected in the added Scope 1 and Scope 3 emissions. For the purposes of post-earthquake PBEE using carbon footprint as a DV, the procedure remains the same as that described above; however, the repair quantities (Qs) are modified by the CO₂ multipliers. Details will be presented in forthcoming publications.

Conclusions

Examples of PBSA and PBSD of typical California reinforced concrete highway bridges are presented in this paper. The performance metrics considered are post-earthquake consequences such as repair costs and repair times, and necessarily involve the development of probabilistic damage, repair, and loss models. Note that while procedures such as displacement-based design are intended to allow design for target performance, there is a very large leap presented in the PBSD example in this paper: performance is specified as a loss level, and the epistemic and aleatory uncertainties are explicitly accounted for, resulting in design at different confidence levels. However, the trade-off is the limited level of complexity captured by the models used.

When evaluating several predetermined design options in parallel (i.e., performing PBSAs), it is possible to use higher fidelity models that include coupled bridge-foundation-ground effects. The ability to investigate several different ground (site) scenarios was illustrated using the enabling software BridgePBEE that would previously have been computationally challenging. These scenarios outlined the possible impacts of neglecting the complete bridge-ground system on PBEE outcomes. The framework used to disaggregate the system in performance groups and reassemble outcomes is robust and capable of handling repair costs/times as well as new efforts on adding the carbon footprint as a performance metric.

Acknowledgments

The PBSD and PBSA results presented in this paper are the culmination of the efforts of many people and years of work. The author would like to gratefully acknowledge several key co-authors and collaborators. John-Michael Wong helped develop the detailed damage and loss metrics that took advantage of the additional repair model in the probabilistic framework. Ahmed Elgamal and Jinchi Lu brought together
bridge-ground modeling with PBSA into the graphical user interface that eventually became BridgePBEE (http://peer.berkeley.edu/bridgepbee/). Finally, current NSF research with Omer Tatari and Ahmed Elgamal is aimed at bringing sustainability metrics to PBEE. Tatari’s contributions are reflected in the paradigm shift and methods discussed in the last section of the paper.

References


